PHYSICS REPORTS

Physics Reports 333–334 (2000) 147–165

www.elsevier.com/locate/physrep

The cosmological matter density

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Abstract

The status of observational cosmology is a subject that David Schramm followed intently. As spokesman for the entire field of particle astrophysics, David was interested in the full picture. He was always conversant with the latest developments in observations of the light elements, as they directly impacted his work on primordial nucleosynthesis and the resulting predicted abundances of deuterium, helium, and lithium. He was especially keen on knowing the status of the latest measurements of the cosmic density parameter, Ω_{m}, as a sufficiently high value, higher than that predicted for primordial nucleosynthesis, motivates the case for a non-baryonic component of dark matter. He had a deep interest in the phenomenology of large-scale structure, as this provides a powerful clue to the nature of the dark matter and the initial fluctuations generated in the early Universe. This review briefly summarizes current techniques for estimation of the density of the Universe. These estimates on a variety of physical scales yield generally consistent results, suggesting that the dark matter, apart from a possible smooth component, is well mixed with the galaxy distribution on large scales. A near consensus has emerged that the matter density of the Universe, Ω_{m}, is a factor of 3–4 less than required for closure. Measures of the amplitude and growth rate of structure in the local Universe are dependent on a degenerate combination of Ω_{m} and the bias b in the observed galaxy distribution. The unknown bias in the galaxy distribution has been a persistent problem, but methods for breaking the degeneracy exist and are likely to be widely applied in the next several years.

PACS: 98.62.Ck; 98.62.Py; 98.65.Dx; 98.80.Es

1. Introduction

The legacy of David Schramm spans a remarkably broad range of topics in astrophysics and the early Universe. As long ago as 1974, David was keenly concerned with fundamental questions, not

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PII: S0370-1573(00)00020-X

only in big-bang nucleosynthesis, but in other fundamental cosmological issues. An early paper [1] in collaboration with Gott, Gunn, and Tinsley is a seminal statement that details the evidence for a Universe with insufficient mass density to reach the critical value required for closure. In the intervening 25 years, the data have improved tremendously, but the basic conclusions remain unchanged. This brief review describes the tools that can be used to address this question. In the space allotted, it is impossible to do justice to the wide variety of current research which bears on the topic, and I apologize in advance to those whose work I have neglected in the discussion below.

Certainly one of the earliest, most natural expectations of inflationary cosmology was the prediction that the curvature of the Universe today is negligibly small, since the enormous expansion flattens out any initial curvature. The simplest model of a flat Universe is one with mass density in ordinary matter equal to the critical value, i.e. $\Omega_m = 1$. However, this simple Einstein–de Sitter cosmology has now convincingly failed to meet a wide variety of experimental tests, several of which we summarize below. In fact, the current consensus appears to be nearly unanimous: if the Universe is cosmologically flat, it must be dominated today by a smooth component of matter completely unclustered with the visible galaxies. The recent supernovae results suggest that the expansion of the Universe has been *accelerating* rather than decelerating in the recent past [2–4], implying that the smooth component has negative pressure, as in a Universe dominated by a cosmological constant Λ or an active scalar field such as quintessence [5]. This is a most astounding situation, as it would imply that the distant Universe will eventually fade from view as objects pass through the event horizon, and that future cosmologists will have many fewer visible galaxies to contemplate!

Progress in observational cosmology in the past decade has been substantial, but the pace of activity is accelerating still and our understanding of fundamental cosmology in 10 years' time should be considerably deeper than at present. The upcoming CMBR satellites MAP and Planck, plus the enormous redshift surveys now underway, 2DF and the SDSS, could yield definitive estimates of all major cosmological parameters, as well as precision measurements of large scale structure (LSS) at the present epoch. These projects will provide definitive information at $z = 1000$ and 0–0.2. Complementing these studies will be large redshift surveys executed on the Keck and VLT telescopes (the DEEP and VIRMOS projects), which promise to provide strong constraints on LSS at $z = 1$. These studies will provide direct evidence of the evolution of LSS, and they have the potential to perform novel new tests of cosmic parameters, including estimation of the "cosmic pressure". The prospects for progress in observational cosmology have never been brighter!

2. Tools for the estimation of mass density

To derive an estimate of the mean mass density of the Universe, astronomers attempt to measure the masses of its visible components. On extragalactic scales, orbital periods are in excess of 10^8 years, so direct estimation of astronomical masses must come by means of inference from rotation curves of isolated galaxies or application of the virial theorem to the observed random velocities within groups and clusters of galaxies. These traditional methods have been complemented by mass estimates derived by assuming that the hot gas emitting X-rays in rich clusters of galaxies is approximately in hydrostatic equilibrium. A third method of estimating the masses of galaxies and clusters of galaxies is to study the gravitational lensing effects of deep gravitational wells, which distort the light path of background galaxies in the nearby field. As bigger and better CCD detectors have become available in recent years, gravitational lensing studies have become much more practical and have blossomed into a major growth industry.

Primordial nucleosynthesis arguments provide an indirect estimate of the baryon density. Deuterium is the most sensitive "baryometer", with an expected abundance that drops sharply with increasing baryon density. But deuterium is fragile and is always destroyed in stars, so it is best to measure the deuterium abundance in clouds which have had little star formation. High-dispersion spectroscopy of high redshift Lyman-α clouds toward distant QSOs has proven to be a very powerful tool for this. Although there has been some controversy, the most definitive observations of two low-metallicity clouds observed with the Keck telescope give abundance estimates consistent with $D/H = 3.4 \pm 0.3 \times 10^{-5}$. With standard big bang nucleosynthesis (BBN) calculations [6], this implies a present-day baryon density in units of the critical density, $\Omega_B h_{100}^2 = 0.019 \pm 0.001$ [7] where Ω_B is the fraction of critical mass density in baryons, and $h_{100} = H_0/100$ (Henceforth we shall write $h = h_{100}$.) Since all evidence suggests $h > 0.5$, the nucleosynthesis constraints imply baryon fraction $\Omega_B < 0.08$. If the total mass density is inferred to be larger than this value, then nonbaryonic matter must be abundant in the Universe. Thus to test cosmological models and set constraints on dark matter, it is of considerable interest to measure the clustered component of the density on all available scales.

2.1. Mass and luminosity density

To begin to assess the mass density of the Universe, one can assume that the light distribution traces the mass distribution and see where that might lead. The observed distribution of galaxy luminosities, $\phi(L)$, when integrated over luminosity, defines the *luminosity density* of the local Universe. Although there is currently an embarassing 50% inconsistency in the overall normalization from one survey to another, the integral over $\phi(L)$ converges and in the B band the integrated density is approximately $L \approx 2 \times 10^8 h M_\odot/\mathrm{Mpc}^3$ [8–10]. Comparing this to the critical mass density of the Universe,

$$\rho_c = H_0^2/(3/8\pi G) ,$$

leads to an average mass to light ratio expressed in solar units,

$$\langle M/L \rangle = 1350 h \Omega_m .$$

The mass associated with luminous baryons has typical $M/L \sim 1$–10, consistent with $\Omega_m < 0.01$. This value is characteristic of the M/L ratio observed in the centers of galaxies or in open and globular star clusters on scales of order 1–100 pc. These regions are dominated by baryons; cooling processes undoubtedly allow baryons to shrink inward, leading to further compression after virialization.

On the scale of 0.1–20 kpc, the observed rotation curves of galaxies combined with the reasonable assumption of nearly circular orbits allow a simple measurement of the interior mass as a function of radius. The ubiquitous flat rotation curves seen in spiral galaxies suggest that the interior mass grows linearly with distance, whereas the interior luminosity distribution converges handily, thus arguing that the M/L ratio grows linearly with distance in the outer halos of nearly all spirals. Elliptical galaxies do not have ordered rotation curves, but in a few favorable cases, prominent X-ray halos provide evidence for a deep, extended potential well generated by unseen matter. Flat rotation curves provide the most direct and potent evidence that dark matter is a component of all galaxies. However, galaxy halos can only be tracked to scales < 50 kpc, and the rotation curves alone give no upper limit to the mass density Ω_m.

The characteristic M/L values are observed to increase linearly with measurement scale up to ≈ 1 Mpc, but on larger scales the situation is confused [11]. Tracking the growth of this M/L curve is central to understanding the dark matter distribution of the Universe. From the observed rotation curves of galaxies, it is clear that the dark matter must be less clustered on small scales than the starlight; the key is to understand the upper limit of the trend of increasing M/L. The growth of M/L continues on the scales of binary pairs of galaxies ($r < 100h^{-1}$ kpc) and small groups of galaxies ($r < 1h^{-1}$ Mpc). On an individual basis, pairs and small groups of galaxies are difficult to study [12] because the virial theorem is a very coarse tool and the definitions of groups are often somewhat ambiguous given the overall filamentary nature of the underlying large-scale structure of the galaxy distribution. Fortunately, these questions of group membership are less ambiguous for the rare rich clusters of galaxies.

2.2. Ω_m Derived from rich clusters of galaxies

Rich clusters of galaxies have for many years played a key role in attempts to weigh the Universe. They are the largest virialized systems in the present Universe; and accordingly rare, with typical intercluster separation of $50h^{-1}$ Mpc. Although they contain hundreds or even thousands of galaxies, only $\approx 5\%$ of observed galaxies are located within rich clusters. Clusters are generally easy to distinguish within projected maps of the galaxy distribution as enhanced surface densities, but only with redshift information can chance superpositions of related or unrelated groups be eliminated as false clusters.

The observed velocity dispersions of galaxies within clusters range from 500 to 1500 km/s. Using the virial theorem, this dispersion, combined with a characteristic size of order $1h^{-1}$ Mpc, leads to M/L ratio estimates (in the B band) in the range 200–400h [11,13], consistent with $\Omega_m \sim 0.15$–0.30 if this M/L value is characteristic of the Universe as a whole. Rich clusters contain a substantial amount of unseen matter, a remarkable result first pointed out by Zwicky some 60 years ago [14]. The high M/L ratio of clusters of galaxies derived from the virial theorem is an important datum, one that is consistent with several complementary estimates, as described below.

With detailed information on the number density and velocity dispersion as a function of distance from the cluster center, one can go beyond the single parameter derived from the virial theorem and use the Jeans' equation of hydrostatic equilibrium to ask whether the surface density of galaxies matches the surface density of the mass [15]. Although there is some sensitivity to the assumed degree of isotropy of the orbits of the galaxies, the best evidence to date suggests that the mass field derived from the blue cluster galaxies is the same as that derived from the red cluster galaxies, and that the M/L ratio as a function of radius is consistent with estimates from virial analysis [15]. That is, there is no evidence that the dark matter is less clustered than the galaxies in the vicinity of the rich clusters.

Rich galaxy clusters are so hot ($T \sim 10^7$–10^8 K, or 1–10 keV) that hydrogen and helium atoms are fully stripped, so inefficient bremstrallung radiation is the dominant radiative loss mechanism.

Thus, except in the cluster cores, the diffuse gas of the clusters cannot cool in the lifetime of the Universe, and the baryon fraction of clusters should therefore be characteristic of the Universe as a whole. Furthermore, the thermal X-ray emission from the hot gas can be imaged by suitable space-based instruments, and the projected mass profile can be directly inferred (assuming hydrostatic equilibrium) from the measured luminosity and temperature profile of the X-ray observations [16]. These mass estimates are, in general, consistent with the M/L estimates derived from virial analysis of the galaxy distribution, but they do not have the Poisson noise limitations of such objects. With the superb imaging properties and simultaneous energy resolving power of the pending Chandra satellite, our understanding of density and temperature profiles of rich galaxy clusters will reach a new much higher level of sophistication within the next few years.

Galaxy clusters are dynamically young, and evidence for merging of subunits and hydrodynamic shocks is seen in the best-studied nearby clusters [17]. These departures from equilibrium complicate mass estimates, but they do conform with the predictions of hierarchical models for the formation of large scale structure. Numerical simulations [18] demonstrate that the hydrostatic assumption works to reasonable precision in spite of the ongoing merger events in the clusters.

Since the diffuse baryons in clusters are observable by their X-ray emission, an amusing test is to consider the cluster mass fraction in baryons compared to BBN predictions [19]. The X-ray emitting gas within $1.5h^{-1}$ Mpc contributes $\approx 6h^{-1.5}\%$ of the cluster virial mass, while the luminous stars contribute another 2%. The cluster baryon fraction is therefore

$$\Omega_B/\Omega_m \geq 0.06h^{-1.5} + 0.02 ,$$

which we write as a lower limit because some of the dark component of the cluster could be baryonic [18]. Combined with the nucleosynthesis estimate for Ω_B, this implies a total matter density $\Omega_m < 0.40$ if $h = 0.5$, and the upper limit drops to $\Omega_m < 0.30$ if $h = 0.75$. Further confirmation of these density estimates is derived via the Sunyaev–Zeldovich effect, in which hot electrons within clusters inverse Compton scatter the photons of the CMBR, causing a dip in intensity in the Rayleigh–Jeans portion of the spectrum. The effect is proportional to the integrated pressure of the cluster in question. Recent interferometric studies in the centimeter band have detected the effect with high significance; an average of results in 18 clusters leads to a constraint of $\Omega_m h = 0.22 + 0.05 - 0.08$, or $\Omega_m = 0.31$ for $h = 0.7$ [20]. To salvage a high estimate for the global mass density, one must argue that the baryon fraction within clusters is higher than the global mean or that the BBN arguments are somehow incorrect.

The potential wells of clusters are so deep that gravitational lensing of nearby background galaxies is very pronounced; long arcs wrapping around the clusters are observed in approximately 60% of objects with with X-ray luminosity $L_x > 10^{45}$ erg/s [21]; distortion of background galaxies is detectable around virtually all rich clusters, and a number of groups are currently investigating reconstruction of the projected mass of the cluster by means of this polarization signal [22–24]. See also [25] for a discussion of the consistency of X-ray and lensing mass estimates of clusters. Kaiser gives a recent review of the weak lensing activities [26]. A very detailed analysis of the $z = 0.42$ supercluster MS0302 + 17 suggests that the projected distribution of early type galaxies (elliptical and lenticular galaxies, comprising only 20% of the overall galaxy population) traces the projected mass inferred from the gravitational shear map with a proportionality equivalent to a constant

$M/L \simeq 260h$, consistent with virial analysis [24]. It seems the more numerous spiral galaxies have little mass associated with them. No trend of M/L with scale is evident from 1.5 to $6h^{-1}$ Mpc, arguing that the clustered component of the dark matter appears to be well mixed with the galaxy distribution on cluster scales and larger. If this is true, then Ω_m cannot be larger than 0.3 and could well be less. If the Kaiser et al. result is confirmed in other studies, it will imply that the dark matter is considerably more clustered than the overall galaxy distribution, which would suggest antibias in the galaxy distribution ($b < 1$) and a very low value of Ω_m.

Yet another use of rich clusters for density estimation is to consider their abundance as a function of redshift. These objects are rare events at the present epoch, and their abundance can therefore be well modeled by the Press–Schechter approach [27]. In the standard inflationary model of fluctuation generation, the initial density fluctuations have a Gaussian distribution, and clusters represent approximately $3-\sigma$ events on a smoothing scale of $8h^{-1}$ Mpc [28]. The local observed abundance of the rich clusters sets a constraint on the amplitude of the mass fluctuations today [28–30], which for a ΛCDM model is given as $\sigma_8 \Omega_m^{0.52-0.13\Omega_m} \approx 0.52 \pm 0.04$, where σ_8 is the rms mass fluctuation on a scale of $8h^{-1}$ Mpc. This obscure normalization is used because the observed clustering amplitude of optically selected galaxies is $\sigma_{8-\mathrm{gal}} = 1$ [31]. We do not know if galaxies are fair tracers of the mass distribution on large scales, and it is customary to characterize a possible bias in the galaxy distribution by the parameter $b = 1/\sigma_8$; $b = 1$ if galaxies trace the large-scale mass distribution. The term "anti-biased" implies $b < 1$. If we live in an Einstein–de Sitter Universe, then cluster abundances argue that the galaxy distribution is much more clustered than the matter, $b \approx 1.9$, whereas the argument implies $b \approx 1.1$ in an $\Omega_\Lambda = 0.7$ flat Universe.

If clusters are very rare today, then at high redshift they should be much rarer still. In a simple Einstein–de Sitter model, a $3-\sigma$ fluctuation today would have been a $6-\sigma$ fluctuation at $z = 1$, whereas open and Λ dominated cosmologies evolve much more slowly in the interval $0 < z < 1$. Thus the abundance of rich clusters at high redshift is very sensitive to the cosmological model, and the degenerate combination of σ_8 and Ω_m can be broken by careful abundance estimates at more than one epoch [32–34]. The current data tentatively favors $\Omega_m \approx 0.2$, $\sigma_8 \approx 1.2$, suggesting a weak antibias. The samples of distant clusters are presently small, but in the next few years we can expect major improvements in our knowledge of the abundance of high redshift galaxy clusters with Chandra and XMM data.

It is important to recall that the abundance test assumes Gaussianity for the primordial mass field. Press–Schechter procedures can be applied to non-Gaussian fields [35,36], which can drastically change the cosmological conclusions. Turning the argument around, the consistency of the cluster abundance constraint on Ω_m with other measurements suggests that the primordial density field must have been approximately Gaussian on cluster scales.

To summarize the arguments on rich cluster masses, there presently exist four distinctly different analyses which all yield consistent conclusions: the galaxy distribution in clusters is essentially unbiased relative to the mass distribution and the density parameter is in the range $0.15 < \Omega_m < 0.3$ if the M/L ratio within clusters is characteristic of the Universe, a possibly dubious assumption. Earlier inconsistent mass estimates derived from lensing versus X-ray imaging or Jeans analyses have largely been resolved as the data quality has improved, and nearly all students of the field are in consensus: there is no support for a closed Universe based on the study of clusters of galaxies.

3. Constraints on Ω_m derived from large-scale structure

A complementary approach to the analysis of distinct clusters and groups of galaxies, is the analysis of mass density derived from the large-scale distribution of galaxies compiled in 2D and 3D catalogs. Statistical inferences can be drawn from such catalogs by a number of means, and the procedures are free from the ambiguous problem of group definition. This section briefly summarizes several methods that have proven useful.

3.1. The shape of the power spectrum

The bias in the galaxy distribution, the degree to which the observed galaxies trace the underlying mass distribution, is a fundamental uncertainty that plagues the field of large-scale structure (LSS). Recent theoretical work on bias and hierarchical clustering has shown [37–40] that on large scales the shape of the galaxy two-point correlation function must be proportional to the matter correlations if the formation of a galaxy is dependent only on *local* conditions, such as density. Thus large-scale features in the observed correlation function $\xi(r)$ or its Fourier transform $P(k)$ should reflect those of the underlying matter fluctuations, even if the galaxy distribution is a biased tracer of the mass.

Within the broad family of cold dark matter models (CDM), the most conspicuous observable feature expected in the LSS is the signature of the horizon scale at the epoch denoting the transition from radiation to matter dominance, which is approximately at $z \approx 2.4\ 10^4 \Omega_m h^2$ [41,42]. This corresponds to a physical length scale today of $d_{eq} = 16.0(\Omega_m h^2)^{-1}$ Mpc. The transformation of the primordial power spectrum during the thermal history of the Universe is denoted by the function $T(k)^2$. For adiabatic initial conditions, scales larger than d_{eq} are unperturbed from the initial conditions presumably imposed in the very early Universe and $T(k)$ approaches unity for small k. On scales smaller than d_{eq}, the suppressed growth of fluctuations during the radiation dominated early phase of the Universe tilts the initial spectrum, leading to an asymptotic behavior $T(k) \propto k^{-2}$ for large k. Since the translation from observed redshift to physical length involves a factor h, the lengthscale denoting the turnover of the power spectrum thus depends on the cosmological combination $\Gamma \equiv \Omega_m h$. A more refined analysis including the effect of baryons on the growth rate of fluctuations gives [43]

$$\Gamma = \Omega_m h \exp[-\Omega_B(1 + \sqrt{2h}/\Omega_m)]\ .$$

In the past decade, our understanding of LSS has undergone a major revolution. Where once the relatively simple, $\Omega_m = 1$, $\Gamma = 0.5$, and $b = 2$, standard cold dark matter (SCDM) model of structure formation seemed very promising [44], we now know that such a framework is hopelessly inconsistent with the observed degree of clustering on scales greater than $20h^{-1}$ Mpc. The most convincing measurement of this deficiency is seen in the angular correlations in the APM catalog [45], which demonstrate that the preferred value is $0.2 < \Gamma < 0.3$. The normalization of fluctuations in the cosmic microwave background radiation (CMBR) [46] is similarly inconsistent with the observed power spectrum of galaxy fluctuations for $\Gamma = 0.5$, but is a much better match if $\Gamma = 0.2$.

A value of the Hubble constant as low as $h = 0.2$ is outside all observed bounds [47,48], so the simplest conclusion is that Ω_m is in the range 0.2–0.5. However, interpretation of the spectral shape parameter Γ is very model dependent, since the thermal history of the Universe could be more complex than naive assumptions would indicate. Possible extensions include a primordial spectral tilt from the expected Harrison–Zeldovich slope of $P(k) \propto k$, isothermal initial conditions, a mixture of hot and cold dark matter, or a decaying neutrino which delayed the onset of the matter dominated era. The CMBR anisotropy as measured by COBE allows many possible models of LSS, but not SCDM, which throughout the 1980s had been the accepted model of large-scale structure development.

3.2. Nonlinear correlation statistics for CDM models

In the local Universe, the observed galaxy two-point correlation function, $\xi(r)$, is a reproducible power law of the form $\xi(r) = (r/r_0)^{-\gamma}$, with $r_0 = 5.1 \pm 0.1 h^{-1}$ Mpc and $\gamma = 1.85 \pm 0.05$ [31,49,50]. The power law extends over the range $0.1 < rh < 10$ Mpc, that is, throughout the non-linear regime of strong clustering and into the quasilinear range. Explanations of the power-law behavior of $\xi(r)$ have never been compelling. Originally the observed slope was argued to be the signature of scale-free Poisson initial conditions ($P(k) \propto k^n$, $n \approx 0$) in an Einstein–de Sitter Universe with isothermal initial conditions [51]. N-body simulations with power law initial conditions show self-similar behavior if $\Omega_m = 1$, but the resulting mass autocorrelation function always shows considerable curvature in the region $100 > \xi(r) > 1$ [52,53]. It has been argued that the observed power-law correlation behavior implies that the galaxy initial conditions of the LSS were not scale free or that galaxies are biased relative to the mass in complicated fashion.

SCDM models are not scale free and have a gradually increasing effective slope n as the non-linear scale advances over time. OCDM and ΛCDM models have steeper mass correlations, because clusters form early, while the linear growth of structure is ongoing, and they then shrink in comoving coordinates as the Universe becomes open and linear growth stops [44]. The overly steep mass correlations of these models is well known; power-law fits to the mass correlation function yield $\gamma > 2$ [54,55]. In these models, the galaxies must be substantially *antibiased* on scales $r < 3h^{-1}$ Mpc in a manner that tracks the non power-law behavior of the mass correlations to yield the smooth power-law of the galaxy correlations. With the recent availability of very high quality N-body simulations, it is apparent that *none* of the currently popular CDM variants have a mass autocorrelation function that is well approximated as a power law [54], and that some sort of scale-dependent bias appears inevitable in this class of cosmogony. One physical explanation for the steep $\xi(r)$ in the ΛCDM model is given by [56]. The key finding is that the M/L ratio of halos has a deep minimum at a halo mass of $\sim 10^{12} M_\odot$; in small halos galaxy formation is inhibited by reheating of the cooled gas, while in large halos the time for hot gas to cool and condense onto individual galaxies becomes excessive. The net mass per L_* galaxy within rich galaxy clusters should thus exceed the mean value.

The study of higher-order correlations in the galaxy distribution has recently had a strong resurgence of interest, largely based on the promise that these statistics have for an independent measure of the bias [57–59]. In the quasilinear regime, $\xi < 1$, second-order perturbation theory implies a simple expression for the reduced skewness S_3 and kurtosis S_4. In a Gaussian distribution, all higher moments vanish, but as the fluctuations approach unity, the positivity constraint and the collapse of overdense structures leads to non-Gaussian behavior. The skewness, for example, in second-order perturbation theory should scale as

$$S_3 = \frac{\langle \delta^3 \rangle}{\langle \delta^2 \rangle^2} \propto b^{-1} \ .$$

Analysis of S_3, S_4, and S_5 based on the APM catalog on a scale of $r = 10h^{-1}$ is consistent with $b = 1$ [60,61], again consistent with low, rather than high values of Ω_{m}.

3.3. Redshift space distortions

Redshift catalogs of galaxies are tabulated lists of the observable combination (in the Newtonian limit)

$$c\left(\frac{\Delta\lambda}{\lambda}\right) = cz = H_0 d + v_{\mathrm{p}} \ ,$$

the sum of the Hubble expansion plus a peculiar velocity deviation from smooth expansion, v_{p}. The peculiar velocity includes large-scale coherent flows induced by the growing large-scale perturbations, as well as small-scale incoherent random thermal motions, such as those within virialized groups and clusters that balance the gravitational energy of potential wells.

Redshift-space distortions are detectable by two distinct means. If the distance d to the object in question can be measured with sufficient precision independently from the redshift, then v_{p} can be measured directly. Astronomers have devised numerous distance estimation techniques, such as the Tully–Fisher correlations for spiral galaxies, $D_n - \sigma$ correlations for lenticular and elliptical galaxies, surface-brightness fluctuations due to the discrete surface density of stars per unit area of a smooth elliptical galaxy, or the apparent brightness of SNeIa explosions. The empirical calibrations for these techniques typically yield fractional distance errors of 20% for Tully–Fisher, but as good as 5% for SNeIa. Since expected values of v_{p} are ~ 600 km/s, only the very local Universe, $v_{\mathrm{p}} < 10{,}000$, is accessible to direct distance estimation; these data sets have been used to measure large-scale flows, which we discuss below.

When only redshifts are available for a sample of galaxies, it is still possible to make statistical inferences about v_{p}. The large-scale structure of the Universe should be statistically isotropic in real space, but observations in *redshift* space are distorted by streaming motions that cause the clustering to be somewhat compressed on large scales [62] and by small-scale thermal motions that elongate virialized structures along the line of sight [42]. The most common estimator of the thermal motions is the pair-weighted velocity dispersion $\sigma_{12}(r)$, which is the rms relative motion in the line of sight of pairs of galaxies as a function of their projected separation [31]. Since rich clusters contain many close pairs of galaxies and have a high velocity dispersion, $\sigma_{12}(r)$ is dominated by these rare objects and is notoriously unstable for small samples [63].

The largest currently available redshift catalog suitable for this type of analysis is that from the Las Campanas redshift survey (LCRS) [64], in which $\sigma_{12}(r) \approx 570 \pm 80$ km/s, for $0.1 < rh < 10$ Mpc [50]. The amplitude of all small-scale thermal velocities will scale as $\sigma_8 \Omega_{\mathrm{m}}^{0.5}$, virtually the same as the constraint based on cluster abundances. That is, the same normalization which yields the observed cluster abundance should lead to a velocity dispersion close to that observed in redshift surveys. However, none of the available cluster-normalized CDM models can match this statistic if the mass points are all weighted equally. In all cluster normalized simulations (i.e. $\sigma_8 \Omega_m^{0.5} \approx 0.5$), the typical predicted line-of-sight pair dispersion is in the range 700–800 km/s for $rh < 1$ Mpc, although the simulation pair dispersion is close to the observed value for larger separations [54]. In a simple prescription that deweights rich clusters, Jing et al. (1998) experiment with variable weighting for the pair analysis, assuming an effective number of galaxies N per cluster of mass M of $N/M \propto M^{-\alpha}$. For modest values, $\alpha \approx 0.08$, they find a much improved fit to $\xi(r)$ and a somewhat improved match to $\sigma_{12}(r)$. This is encouraging but not yet compelling. Statistical procedures that are not so excessively weighted toward cluster centers [65] will hopefully lead to a better understanding of the degree to which antibias can improve the match of the pair statistics.

3.4. The cold local flow

It is an observed fact that the peculiar velocity field around the local group of galaxies is extremely cold, with an absence of blue-shifted galaxies outside the immediate LG itself (excepting toward the direction of the core of the nearby Virgo cluster). The rms v_p is estimated to be 60 km/s for galaxies in the shell $1 < r < 5$ Mpc [66,67]. Attempts to simulate "local groups" within realistic cosmological simulations have failed to generate such cold flows for any cosmological models currently under discussion [68], a problem frequently dubbed the "cosmic mach number dilemma" [69]. Blueshifted galaxies would be trivial to spot from spectroscopic data alone, so their abundance in the simulations and absence in the local Universe is difficult to reconcile. Why are the large-scale streaming motions so large while the local thermal velocities are so cold? We do not at present know if this is a fundamental problem of dark matter, or if the existing census of matter about the LG is somehow biased cold relative to the mean. But low peculiar velocities are a signature of low mass density, and the local flow is one further argument against a high mass density.

4. Mass estimation on scales of $50h^{-1}$ Mpc

All the arguments thus far are consistent with each other and with expectations of a low-density Universe. Their chief weakness is that they are insensitive to a component of the Universe that is unclustered on scales of $\sim 1h^{-1}$ Mpc or to a class of galaxies that is underrepresented within LSS surveys, such as low surface-brightness galaxies. Since we have no clear understanding of the nature of the ubiquitous dark matter, one cannot assume that its clustering properties match those of galaxies on any scale. A component of the Universe that is truly spatially uniform cannot be detected by any local tests, but a component that acts thermally "hot" might cluster on the largest scales. Clusters of galaxies are the largest scales for which one can apply virial arguments, but there are several methods that are applicable for mass estimation on larger scales.

The first of these is deep imaging of large, randomly chosen fields of the sky to measure the gravitational lensing shear induced by the mass distribution along the line of sight. This technique is only now becoming practical, as the new generation of large-field CCD cameras comes into use [23,26]. Several groups are beginning to assault this problem, as it has the potential of providing independent estimates of the amplitude of the *mass* fluctuations on scales of 1–10h^{-1} Mpc. The expected lensing signal is weaker than that induced by rich clusters, but with careful control of systematics and high-quality imaging, it should be observable.

4.1. Large-scale flow fields

A second method is to study the large-scale flow field v_p in our local Universe, generally for galaxies with redshifts $cz < 10{,}000$ km/s. There now exist substantial catalogs of galaxies with Tully–Fisher distance estimates [70,71]. Although the peculiar velocity for an individual galaxy is a noisy datum, the statistics of thousands of objects allows one to infer the large-scale flow patterns of the local galaxy distribution. If these flows have been generated by the growth of large-scale structure, then they should correlate strongly with the *gravity* field $\boldsymbol{g}(\boldsymbol{r})$ of the local inhomogeneous mass distribution [49]. At late times in linear theory, the result is exceedingly simple:

$$\boldsymbol{v}_p(\boldsymbol{r}) = ((2/3\Omega_m^{0.4}H_0))\,\boldsymbol{g}(\boldsymbol{r})\ ,$$

which essentially says that the peculiar velocity equals the peculiar gravity times the available time.

One estimate of peculiar velocity is exceedingly well constrained. The dipole anisotropy of the CMBR measures the motion of the Milky Way relative to the preferred frame of the Universe; in the frame of the local group (LG) of galaxies, the measured anisotropy implies $v_{LG} = 620$ km/s toward $l = 275°$, $b = 27°$. A velocity this large cannot be a primordial residual, since initial peculiar velocities decay as the Universe expands. If the velocity has been induced by the local inhomogeneous mass distribution, how large is the coherence length comoving with the local group? Can we find the mass distribution responsible for this motion?

The subject of large-scale flows only came into existence with the advent of full-sky redshift surveys of galaxies made possible by the NASA/IRAS satellite, which mapped nearly the entire sky in four infrared bands in 1983 and detected approximately 15,000 galaxies [72]. Selection by IRAS flux has the advantages of uniformity over most of the sky and negligible galactic foreground extinction. IRAS-selected galaxies are dusty, star-forming objects; thus an IRAS selected catalog is dominated by spiral galaxies and undersamples rich clusters of galaxies which are dominated by lenticular and elliptical galaxies. IRAS catalogs of galaxies flux-limited at 60 μm wavelength cover $\approx 85\%$ of the sky. Until last year, the largest available catalog was the 1.2 Jy catalog [73], which contains 5300 galaxies with a median redshift of 6000 km/s. The PSCZ survey, a catalog of 13,000 galaxies flux limited to 0.6 Jy and with median redshift 8000 km/s, is now complete [74]. An optically selected catalog, supplemented with IRAS galaxies in low galactic latitude regions, ORS, has also recently become available [80] and provides a useful check on the effect of the missing early type galaxies within the IRAS catalogs.

The nearly full-sky nature of these redshift catalogs enabless the construction of gravity maps with a variety of algorithms that compensate for redshift distortions [75–77]. If the large scale galaxy distribution even approximately traces the underlying mass field, then the peculiar velocity and gravity fields should be aligned, with an amplitude that is proportional to $\Omega_m^{0.6}$. If the galaxy distribution is linearly biased with respect to the mass, then the fields should still align, but only the degenerate combination $\beta \equiv \Omega^{0.6}/b$ will be measurable. This is virtually the same combination that determines the cluster abundance and the amplitude of the thermal velocities, but it applies on a much larger scale.

The simplest application of such gravity maps is to examine the question of the CMBR dipole. The cumulative gravity derived from the PSCZ and 1.2 Jy catalogs as a function of limiting redshift shows near convergence of the acceleration at $cz = 4000$ km/s, with both catalogs giving very similar results for $cz < 10{,}000$ km/s [78]. There is continued, weak growth of the dipole on scales greater than 10,000 km/s, but the gravity maps strongly suggest that the region comoving with the local group is confined to those galaxies with $cz < 1000$–2000 km/s, and that there should be strong shear in the measured large-scale flow field for galaxies with $cz > 4000$ km/s. The misalignment of the gravitational dipole relative to the CMBR dipole is remarkably small; in the PSCZ survey, which supercedes the older 1.2 Jy survey, these vectors are misaligned by only 15°, a remarkable result that strongly suggests the large-scale flows have indeed been generated by the inhomogeneous mass distribution. This level of misalignment is consistent with the Poisson shot noise of the dilute sampling of the PSCZ catalog, and it constrains the impact of non-linear flows for which the velocity vector need not align with the gravity vector. For example, the unknown transverse motion of the Milky Way relative to our neighbor M31 affects the transformation to the local group frame. The small misalignment suggests that this transverse motion is less than 100 km/s.

The overall amplitude of the gravity dipole is proportional to β, but the large-scale convergence of the dipole is difficult to prove, and constraints on β tend to be weak [78–80]. Stronger constraints are possible with detailed comparison of peculiar velocities and gravity for a large set of points. These comparisons have been done with a variety of techniques and data sets over the past decade [75,77,81–83]. Examples of the comparison between the inferred gravitational field versus the observed velocity field are shown in Figs. 1 and 2 [83,84]. Note how gravity maps based on either the IRAS or ORS catalogs [85] are in qualitative agreeement with the measured peculiar velocity field, a spectacular confirmation that gravitational instability is responsible for the growth of structure in the Universe. The flow pattern (in the LG frame) is dominated by a dipole signal which is the reflex of the motion of the LG. The SNeIa comparison is limited to a sample of only 24 objects, and sets an upper limit of $\beta < 0.7$. The nearby SN distances suffer completely different systematics from Tully–Fisher data, and it is encouraging that the different data sets match so well. This result will continue to improve in coming years as more nearby SNe are discovered.

Unfortunately, the results of such flow analyses have been contradictory, and the technique has so far not lived up to its potential. The method gives us a unique, model-independent handle on mass measurement on the scale which has generated the CMBR dipole signature, $\sim 50h^{-1}$ Mpc, and the estimate of β that could be inferred from the velocity-gravity comparison does not suffer from cosmic variance although it is bedeviled by large-scale bias. However, the measurement of peculiar velocities is fraught with systematic errors, since it has been extremely difficult to assemble fully homogeneous catalogs of distances over the full sky. The comparison of the Mark III catalog with the IRAS 1.2 Jy survey [82,83] yields unphysical residuals that preclude a believable determination of β. The SFI catalog [86] compared to IRAS has a better residual map; it gives an estimate $\beta = 0.6 \pm 0.1$. A very sophisticated likelihood analysis that properly treats triple-valued regions yields a strong constraint, $\beta = 0.5 \pm 0.04$ [82], but does not assess whether the IRAS flow model is an acceptable description of the observed flows. In contradiction to the relatively low values of β inferred from the gravity–velocity field comparisons are the consistently high values of β extracted from the POTENT procedure. POTENT is an algorithm which compares the divergence of the observed peculiar velocity field to the smoothed IRAS density map [87]. Using

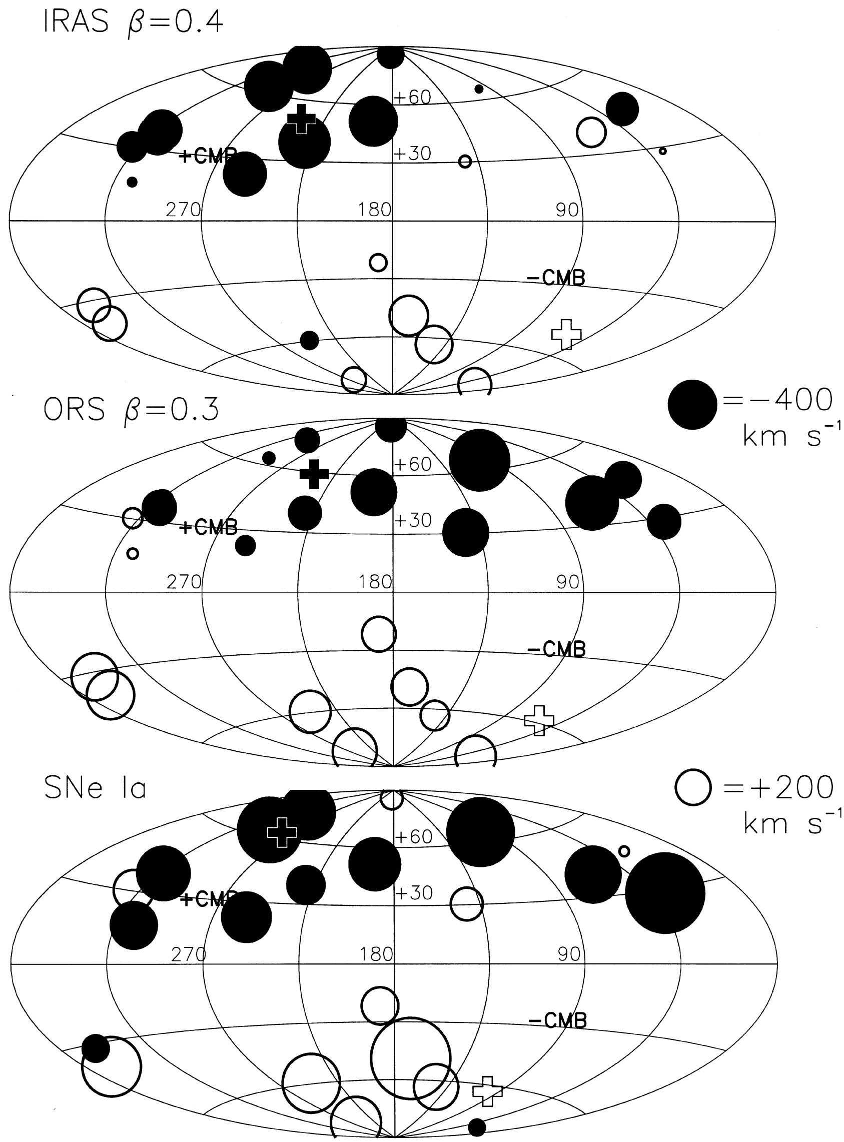

Fig. 1. The peculiar velocity of nearby SNeIa events ($cz < 10{,}000$ km/s) distributed on the sky (84). Note the strong reflex dipole signature and that the gravity maps, based on either the IRAS or ORS catalogs (85), are in excellent agreement with the observed peculiar velocities.

the same Mark III catalog for which the $v-g$ comparison yields $\beta = 0.5$, POTENT yields $\beta = 0.89 \pm 0.15$ [88], a 2.5 σ discrepancy. Until all methods give consistent values of β, no results should be taken too seriously. The reasons for the discrepancy of POTENT with other analyses are not clear, but the treatment of the data is quite different for POTENT. By taking a divergence of a carefully filtered, noisy set of data, the POTENT method gives most of its weight to the highest spatial frequencies remaining in the velocity field. But the $v-g$ comparisons give most of their

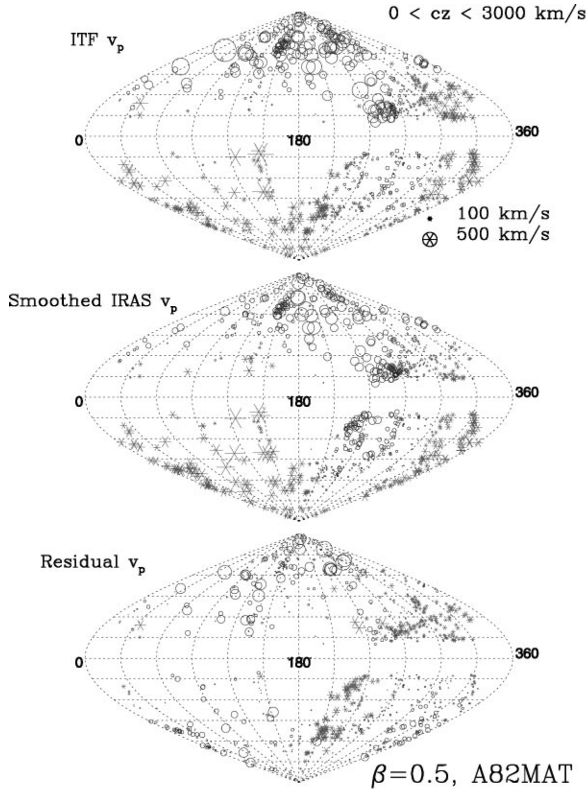

Fig. 2. The peculiar velocities of 900 galaxies with $cz < 3000$ km/s from the Mark III catalog. Circles denote galaxies with $v_p < 0$, while stars denotes those with $v_p > 0$ (LG frame). The size of each symbol is proportional to the amplitude of the flow. Note the excellent qualitative agreement between this nearby flow pattern and that expected for the same galaxies, based on the 1.2 Jy IRAS gravity map.

weight to the lowest spatial frequencies in the fields, the same components which describe the reflex of the motion of the LG. If the velocity and gravity fields are inconsistent with each other, then different fit results with very different weighting functions are not unexpected.

On larger scales, even more problems emerge. The analyses of flow for $cz > 6000$ km/s are inconsistent with everything we have learned on smaller ones. The Lauer–Postman report of a bulk flow coherent on scales of $100h^{-1}$ [89] is inconsistent with everything inferred from the IRAS maps and is orthogonal to the CMBR dipole. More recent peculiar velocity measurements on a similar scale [90,91] again suggest large coherent flow, but now perpendicular to the Lauer–Postman result and in the direction of the CMBR dipole. The more recent analyses would imply that everything with $cz < 10{,}000$ is coherently flowing with the LG rather than being the source of its motion! Not all of these results can be correct!

Since the full-sky IRAS maps are almost perfectly isotropic for redshifts larger than 6000 km/s and yet have a gravitational dipole signature extremely well aligned with the CMBR dipole, they will inevitably predict that $\frac{2}{3}$ of the CMBR dipole of the LG is generated by material much closer than this distance. Therefore, in the LG frame, one expects to see a strong reflex dipole in v_p for galaxies at this distance and greater. This is the key feature of the predicted maps that dominates the field at large distance, but none of the more distant flow studies detect a large reflex. If the reflex dipole is truly absent in the peculiar velocity field at $cz \approx 10{,}000$ km/s, then the mass of the interior matter must be very low, and β must be very small, much smaller than inferred by comparison to the Mark III or SFI catalogs at redshifts $cz < 6000$ km/s.

4.2. Velocity power spectra

A third possible approach to measuring Ω_{m} on large scale is to examine the velocity power spectrum without reference to the IRAS gravity maps [92]. Here one fits a parameterized shape of the power spectrum $P(k)$ to the observed amplitude of the peculiar velocities. The technique is attractive, in principle, since the velocities are a response to the mass fluctuations which might be distributed on scales larger than those densely probed by the IRAS galaxy catalogs. The inferred large-scale bulk flows smoothed on scales of $\sim 50h^{-1}$ Mpc are estimated to be $v_{\mathrm{bulk}} \approx 350$ km/s, which requires substantial large-scale power, more than is typical in SCDM models but consistent with rms expectations in ΛCDM models [54]. The method sets a constraint on $P(k)\Omega_{\mathrm{m}}^{1.2}$ and is equivalent to a normalization of $\sigma_8 \Omega_{\mathrm{m}}^{0.6} = 0.82 \pm 0.12$, considerably higher than that favored by the cluster normalization or the velocity–gravity field comparison.

There are a number of potential difficulties with the velocity power-spectrum approach. In contrast to the velocity–gravity field comparison, a first-order procedure that keeps phase information and does not suffer from cosmic variance, the velocity–power likelihood analysis discards phase information and simply compares the measured power to that expected in various models. Each mode in the power estimate is exponentially distributed, and the number of independent modes is small, so that the expected cosmic variance is large in available catalogs. Furthermore, aliases of small-scale thermal velocities also contribute to long-wavelength modes. However, the most serious difficulty with velocity–power studies is controlling large-scale systematic errors in the peculiar velocity catalog. Any non-uniformity in the catalog translates to excess variance, and large-scale calibration non-uniformities in the Tully–Fisher data will be buried in the signal of the bulk flows. The velocity–power spectrum is dominated by the longest wavelength modes, but because the peculiar velocity errors are always a fixed fraction of the redshift of the galaxy, the longest modes will always be least secure. The confidence in peculiar velocity fields of any sort must fall linearly with redshift, so conclusions based on the foreground of a peculiar-velocity catalog should be more secure than those derived from the background of the same catalog. My personal preference would therefore be to treat this recent result as an *upper limit* to the fluctuation normalization.

4.3. Mean streaming

A fourth approach is to address the mean relative streaming velocity of pairs of galaxies as a function of their separation. If structure is growing by means of gravitational instability, then pairs of galaxies must, on average, be moving towards each other. The conservation of pairs equation emerging from the BBGKY hieararchy can be analyzed on both linear and non-linear scales; a simple expression that is quite accurate and which fits N-body simulations has been recently derived [93]. Application of this measure to the Mark III catalog of peculiar velocities

[94,95] on scales $r < 10h^{-1}$ Mpc cleanly detects the streaming at a value consistent with low density, $\Omega_m = 0.35 \pm 0.25$. Because this is a non-linear test, it should eventually be possible to break the degeneracy between Ω_m and σ_8. Like all tests of peculiar velocity, the estimate is dominated by foreground galaxies, which have the smallest errors, and therefore the available volume to execute the test is presently rather small.

4.4. Conclusions from large-scale measurements

Given the current confusion of contradictory results on large scales, any conclusion must be considered as provisional. The qualitative comparison of the gravity and velocity fields for the galaxy distribution within 6000 km/s shows basic consistency and is a strong confirmation of the overall paradigm of structure formation via gravitational instability. If there exists an abundance of faint galaxies not normally detected in flux-limited surveys, they are either correlated with the higher surface-brightness galaxies or their mass density is negligible. But quantitative conclusions regarding the mass density are premature, as calibration problems probably lurk in the peculiar velocity catalogs. The flow analyses are all based on linear perturbation theory and linear biasing, resulting in a degeneracy between b and Ω_m. Attempts to break the degeneracy by non-linear effects must also include second-order terms in the bias, which are unknown. All the β values given above are based on the IRAS catalog, which has a correlation length smaller than that of optically selected galaxies. In fact, $\sigma_{8-\mathrm{iras}}/\sigma_{8-\mathrm{opt}} = 0.7$ [96]. For $\beta_{\mathrm{iras}} \approx 0.5$–$0.6$, this yields $\beta_{\mathrm{opt}} \sim 0.35$–$42$, or $\sigma_8 \Omega_m^{0.6} = 0.35$–$42$, assuming the galaxy bias is linear on large scales. For unbiased optical galaxies, this translates to a rather low estimate, $\Omega_m = 0.17$–0.24. This range of Ω_m is consistent with the density estimates based on the recent estimates derived from the mean streaming analysis [94]. Since these estimates are so similar to those derived for clusters of galaxies, we must conclude that, with the possible exception of a completely smooth component, the dark matter of the Universe fully participates in clustering on the scale of rich clusters of galaxies. All currently fashionable models of cosmogony are consistent with this observation.

This normalization is slightly lower than that derived from cluster abundances and more than a factor of two smaller than that inferred from POTENT or velocity–power spectral analysis. If the amplitude of fluctuations were as large as suggested by those techniques, the velocity shear within 6000 km/s should be twice as large as is measured, unless the mass per galaxy grows with distance from the Milky Way. This seems precluded by the observed shear of the nearby SNe and by the direct comparisons to Mark III and SFI. Thus considerable work remains to clarify the various inconsistencies!

5. Conclusions

This brief, idiosyncratic review of the current estimates of the density parameter Ω_m has highlighted selected fields of current research. As described above, there is an emerging consensus that Ω_m is considerably less than unity, with best estimates $0.15 < \Omega_m < 0.35$ [97]. Based on luminosity density arguments and the observed mass to light ratio of clusters of galaxies, Schramm and collaborators 25 years ago [1] had already reached this conclusion; in the intervening years the data have improved enormously but the best estimates for Ω_m have barely budged. The normalization of clustering, $\sigma_8 \Omega_m^{0.5}$, when measured by rich cluster abundances is slightly higher than that suggested by the velocity measurements, either based on small-scale thermal motions or on the large-scale flows, although this latter point remains controversial. The status of the "temperature" of the Universe remains somewhat uncertain. On small scales, the velocity dispersion of galaxies appears to be very cold, and achieving such a low mean "temperature" seems to require low values of the LSS normalization constant, antibias toward cluster centers, or both.

As there has emerged a consensus for the ΛCDM Universe [97,98], further tests of the consequences of such models have great importance. The small-scale antibias, or extra concentration of mass within cluster centers, is one prediction, and perhaps it has already been detected, as discussed above [24]. The critical tests of this prediction are likely to become definitive within the next few years, given the rapid progress in weak lensing studies.

In the next few years much tighter constraints on all cosmological parameters will emerge from CMBR studies, from weak lensing studies of field galaxies, from precision measurements of LSS courtesy of the SDSS and 2DF surveys, and from high redshift studies such as the Keck/DEEP and VLT/VIRMOS projects. Cosmology is now a data-rich subject, and within the next decade we shall have multiple probes that must eventually lead to a consistent understanding of all the cosmological parameters.

Acknowledgements

I thank Jeff Newman for a careful reading of the manuscript. This work was supported by NSF grant AST95-28340 and by NASA contract NAG5-7833.

References

[1] J.R. Gott, J.E. Gunn, D.N. Schramm, B.M. Tinsley, Astrophys. J. 194 (1974) 543.
[2] S. Perlmutter, G. Altering, G. Goldhaber, G.A. Knop et al., astro-ph/9812133, 1998.
[3] P.M. Garnavich, S. Jha, P. Challis, A. Clocchiatti et al., Astrophys. J. 509 (1998) 74, astro-ph/9806396.
[4] R. Kirshner, Proc. Natl. Acad. Sci. 96 (1999) 4224.
[5] P.J. Steinhardt, R.R. Caldwell, in: Y.I. Byun, K.W. Ng (Eds.), Cosmic Microwave Background and Large Scale Structure of the Universe. ASP Conference Series, Vol. 151, 1998, p. 13.
[6] T.P. Walker, G. Steigman, H. Kang, D.M. Schramm, K.A. Olive, Astrophys. J. 376 (1991) 51.
[7] S. Burles, D. Tytler, astro-ph/9803071, 1998.
[8] G. Efstathiou, R.S. Ellis, B.A. Peterson, Mon. Not. R. Astron. Soc. 232 (1988) 431.
[9] J. Loveday, B.A. Peterson, G. Efstathiou, S.J. Maddox, Astrophys. J. 390 (1992) 338L.
[10] H. Lin, R.P. Kirshner, S.A. Shectman, S.D. Landy, A. Oemler, D.L. Tucker, P.L. Schechter, Astrophys. J. 464 (1996) 60.
[11] N.A. Bahcall, L.M. Lubin, V. Dorman, Astrophys. J. 447 (1995) L81.
[12] A. Diaferio, M. Ramella, M.J. Geller, A. Ferrari, Astron. J. 105 (1993) 2035.
[13] R.G. Carlberg, H.K. Yee, E. Ellingson, R. Abraham, S.L. Morris, C.J. Pritchet, Astrophys. J. 462 (1996) 32.
[14] F. Zwicky, Helv. Phys. Acta 6 (1933) 110.
[15] R.G. Carlberg, H.K. Yee, E. Ellingson, S.L. Morris et al., Astrophys. J. 485 (1997) L13.
[16] C. Sarazin, X-ray Emission from Clusters of Galaxies, Cambridge University Press, Cambridge, 1988.
[17] K. Roettiger, J.M. Stone, R.F. Mushotsky, Astrophys. J. 493 (1998) 62.
[18] A.E. Evrard, Mon. Not. R. Astron. Soc. 292 (1997) 289.

[19] S.D.M. White, J.F. Navarro, A.E. Evrard, C.S. Frenk, Nature 366 (1993) 429.
[20] L. Grego, J.E. Carlstrom, M.K. Joy, E.D. Reese, G.P. Holder, S. Patel, A.R. Cooray, W.L. Holzapfel, astro-ph/0003085, Astrophys. J. (1999) submitted.
[21] G.A. Luppino, I.M. Gioia, F. Hammer, O. Le Fevre, J.A. Annis, astro-ph/9812355.
[22] J.A. Tyson, F. Valdes, J.F. Jarvis, A.P. Mills, Astrophys. J. 281 (1984) 59.
[23] P. Schneider, L. van Waerbeke, B. Jain, G. Kruse, Mon. Not. R. Astron. Soc. 296 (1998) 873.
[24] N. Kaiser, G. Wilson, G. Luppino, L. Kofman, I. Gioia, M. Metzger, H. Dahle, astro-ph/9809268 (1998).
[25] S.W. Allen, Mon. Not. R. Astron. Soc. 296 (1998) 392.
[26] N. Kaiser, in: S. Colombi, Y. Mellier, B. Raban (Eds.), Wide Field Surveys in Cosmology, Editions Frontières, Dreux, 1998, p. 5.
[27] W.H. Press, P. Schechter, Astrophys. J. 187 (1974) 425.
[28] S.D.M. White, G. Efstathiou, C.S. Frenk, Mon. Not. R. Astron. Soc. 262 (1993) 1023.
[29] V. Eke, S. Cole, C.S. Frenk, Mon. Not. R. Astron. Soc. 282 (1996) 263.
[30] L. Wang, P. Steinhardt, Astrophys. J. 508 (1998) 483.
[31] M. Davis, P.J.E. Peebles, Astrophys. J. 267 (1983) 465.
[32] V.R. Eke, S. Cole, C.S. Frenk, P. Henry, Mon. Not. R. Astron. Soc. 298 (1998) 1145.
[33] N. Bahcall, X. Fan, Astrophys. J. 504 (1998) 1.
[34] J.P. Henry, Astrophys. J. 489 (1997) L1.
[35] J. Robinson, J. Baker, astro-ph/9905098, 1999.
[36] J.A. Willick, astro-ph/9904367, 1999.
[37] P. Coles, Mon. Not. R. Astron. Soc. 262 (1993) 1065.
[38] D. Munshi, P. Coles, A. Melott, Mon. Not. R. Astron. Soc., submitted (1999) astro-ph/9902215.
[39] F. Bernardeau, R. Schaeffer, Astron. Astrophys. 349 (1999) 697, astro-ph/9903387.
[40] P. Coles, A. Melott, D. Munshi, astro-ph/9904253, 1999.
[41] P.J.E. Peebles, Principles of Physical Cosmology, Princeton University Press, Princeton, NJ, 1992.
[42] J. Peacock, Cosmological Physics, Cambridge University Press, Cambridge, 1998.
[43] N. Sugiyama, Astrophys. J. Suppl. 100 (1995) 281.
[44] M. Davis, G. Efstathiou, C. Frenk, S.D.M. White, Astrophys. J. 292 (1985) 371.
[45] S.J. Maddox, G. Efstathiou, W.J. Sutherland, J. Loveday, Mon. Not. R. Astron. Soc. 242 (1990) 43P.
[46] E. Bunn, M. White, Astrophys. J. 480 (1997) 6.
[47] A. Sandage, G.A. Tammann, in: N. Turok (Ed.), Critical Dialogues in Cosmology, World Scientific, Singapore, 1997, p. 130.
[48] W.L. Freedman, in: N. Turok (Ed.), Critical Dialogues in Cosmology, World Scientific, Singapore, 1997, p. 92.
[49] P.J.E. Peebles, Large-Scale Structure in the Universe, Princeton University Press, Princeton, NJ, 1980.
[50] Y.P. Jing, H.J. Mo, G. Boerner, Astrophys. J. 494 (1998) 1.
[51] P.J.E. Peebles, Astrophys. J. 189 (1974) L51.
[52] G. Efstathiou, C.S. Frenk, S.D.M. White, M. Davis, Mon. Not. R. Astron. Soc. 235 (1988) 715.
[53] S. Colombi, F.R. Bouchet, L. Hernquist, Astrophys. J. 465 (1996) 14.
[54] A. Jenkins, C.S. Frenk, F.R. Pearce, P.A. Thomas, J.M. Colberg, S.D.M. White, H.M.P. Couchman, J.A. Peacock, G. Efstathiou, A.H. Nelson, astro-ph/9709010, 1997.
[55] M.A.K. Gross, R.S. Somerville, J.R. Primack, J. Holtzman, A. Klypin, Mon. Not. R. Astron. Soc. 301 (1998) 81.
[56] A.J. Benson, S. Cole, C.S. Frenk, C.M. Baugh, C.G. Lacey, astro-ph/9903343, 1999.
[57] J. Fry, Astrophys. J. 461 (1996) L65.
[58] R. Scoccimarro, S. Colombi, J.N. Fry, J.A. Frieman, E. Hivon, A. Melott, Astrophys. J. 496 (1998) 586.
[59] E. Gaztanaga, F. Bernardeau, Astron. Astrophys. 331 (1998) 829.
[60] Y.P. Jing, astro-ph/9710188, 1997.
[61] E. Gaztanaga, Mon. Not. R. Astron. Soc. 268 (1994) 913.
[62] N. Kaiser, Mon. Not. R. Astron. Soc. 227 (1987) 1.
[63] H.J. Mo, Y.P. Jing, G. Boerner, Mon. Not. R. Astron. Soc. 264 (1993) 825.
[64] S.A. Shectman, S.D. Landy, A. Oemler, D.L. Tucker, H. Lin, R.P. Kirshner, P.L. Schechter, Astrophys. J. 470 (1996) 172.

[65] M. Davis, A. Miller, S. White, Astrophys. J. 490 (1997) 63.
[66] A. Sandage, Astrophys. J. 307 (1986) 1.
[67] P.J.E. Peebles, Astrophys. J. 332 (1988) 17.
[68] F. Governato, B. Moore, R. Cen, J. Stadel, G. Lake, T. Quinn, New Astronomy 2 (1997) 91.
[69] J.P. Ostriker, Y. Suto, Astrophys. J. 348 (1990) 378.
[70] J.A. Willick, S. Courteau, S.M. Faber, D. Burstein, A. Dekel, T. Kolatt, Astrophys. J. 457 (1996) 460.
[71] R. Giovanelli, in: S. Colombi, Y. Mellier (Eds.), Wide Field Surveys in Cosmology, Editions Frontières, Dreux, 1998, p. 109.
[72] C.A. Beichman, G. Neugebauer, H.J. Habing, P.E. Clegg, T.J. Chester (Eds.), IRAS Catalogs and Atlases: Explanatory Supplement, US. Govt. Printing Office, Washington DC, 1988.
[73] K.B. Fisher, J.P. Huchra, M.A. Strauss, M. Davis, A. Yahil, D. Schlegel, Astrophys. J. (Suppl.) 100 (1995) 69.
[74] W. Saunders, S. Oliver, O. Keeble, M. Rowan-Robinson et al., in: S. Colombi, Y. Mellier, B. Raban (Eds.), Wide Field Surveys in Cosmology, Editions Frontieres, Dreux, 1998, p. 71.
[75] A. Yahil, M.A. Strauss, M. Davis, J.P. Huchra, Astrophys. J. 372 (1991) 380.
[76] A. Nusser, M. Davis, Astrophys. J. 421 (1994) 1.
[77] A. Nusser, M. Davis, Mon. Not. R. Astron. Soc. 276 (1995) 1391.
[78] I. Schmoldt, E. Branchini, L. Teoddore, G. Efstathiou, C. Frenk, O. Keeble, S. Maddox, S. Oliver, M. Rowan-Robinson, W. Saunders, S.D.M. White, astro-ph/9901087.
[79] N. Vittorio, R. Juszkiewicz, M. Davis, Nature 323 (1986) 132.
[80] M.A. Strauss, A. Yahil, M. Davis, J. Huchra, K. Fisher, Astrophys. J. 83 (1992) 29.
[81] J.A. Willick, M.A. Strauss, A. Dekel, T. Kolatt, Astrophys. J. 486 (1997) 689.
[82] J.A. Willick, M.A. Strauss, Astrophys. J. 507 (1998) 64.
[83] M. Davis, A. Nusser, J. Willick, Astrophys. J. 473 (1996) 22.
[84] A.G. Riess, M. Davis, J. Baker, R. Kirshner, Astrophys. J. 488 (1997) 1.
[85] J.E. Baker, M. Davis, M.A. Strauss, O. Lahav, B.X. Santiago, Astrophys. J. 508 (1998) 6.
[86] L. da Costa, W. Freudling, G. Wegner, R. Giovanelli, M. Haynes, J. Salzer, G. Wegner, Mon. Not. R. Astron. Soc. 299 (1998) 425.
[87] A. Dekel, Annu. Rev. Astron. Astrophys. 32 (1994) 371.
[88] Y. Sigad, A. Eldar, A. Dekel, M.A. Strauss, A. Yahil, Astrophys. J. 495 (1998) 516.
[89] T.R. Lauer, M. Postman, Astrophys. J. 425 (1994) 418.
[90] M.J. Hudson, R.J. Smith, J.R. Lucey, D.J. Schlegel, R.L. Davies, Astrophys. J. 512 (1999) L79.
[91] J.A. Willick, astro-ph/9812470, 1998.
[92] W. Freudling, I. Zehavi, L. da Costa, A. Dekel, A. Eldar, R. Giovanelli, M. Haynes, J. Salzer, G. Wegner, S. Zaroubi, Astrophys. J. 523 (1999) 1, astro-ph/9904118.
[93] R. Juszkiewicz, V. Springel, R. Durrer, Astrophys. J. Lett. 518 (1999) L25, astro-ph/9812387.
[94] R. Juszkiewicz, P.G. Ferreira, H.A. Feldman, A.H. Jaffe, M. Davis, astro-ph/0001041.
[95] P.G. Ferreira, R. Juszkiewicz, H. Feldman, M. Davis, A.H. Jaffe, Astrophys. J. Lett. 515 (1999) L1.
[96] K.B. Fisher, M. Davis, M.A. Strauss, A. Yahil, J.P. Huchra, Mon. Not. R. Astron. Soc. 266 (1994) 219.
[97] L. Wang, R.R. Caldwell, J.P. Ostriker, P.J. Steinhardt, Astrophys. J. 530 (2000) 17, astro-ph/9901388.
[98] M.S. Turner, Phys. Scr., astro-ph/9901109 (1999).