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Inflation and eternal inflation[☆]

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Abstract

The basic workings of inflationary models are summarized, along with the arguments that strongly suggest that our universe is the product of inflation. The mechanisms that lead to eternal inflation in both new and chaotic models are described. Although the infinity of pocket universes produced by eternal inflation are unobservable, it is argued that eternal inflation has real consequences in terms of the way that predictions are extracted from theoretical models. The ambiguities in defining probabilities in eternally inflating spacetimes are reviewed, with emphasis on the youngness paradox that results from a synchronous gauge regularization technique. Vilenkin's proposal for avoiding these problems is also discussed. © 2000 Elsevier Science B.V. All rights reserved.

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1. Introduction

There are many fascinating issues associated with eternal inflation, so I can think of no subject more appropriate to discuss in a volume commemorating David Schramm. The shock of Dave's untimely death showed that even the most vibrant of human lives is not eternal, but his continued influence on our entire field proves that in many ways David Schramm is truly eternal. Dave is largely responsible for creating the interface between particle physics and cosmology, and is very

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much responsible for cementing together the community in which this interface developed. His warmth, his enthusiasm, and the efforts that he made to welcome young scientists to the field have strengthened our community in a way that will not be forgotten.

I will begin by summarizing the basics of inflation, including a discussion of how inflation works, and why many of us believe that our universe almost certainly evolved through some form of inflation. This material is not new, but I think it should certainly be included in any volume that attempts to summarize the important advances that Dave helped to develop and promote. Then I will move on to discuss eternal inflation, attempting to emphasize that this topic has important implications, and raises important questions, which should not be dismissed as being metaphysical.

2. How does inflation work?

The key property of the laws of physics that makes inflation possible is the existence of states of matter that have a high-energy density which cannot be rapidly lowered. In the original version of the inflationary theory [1], the proposed state was a scalar field in a local minimum of its potential energy function. A similar proposal was advanced by Starobinsky [2], in which the high-energy density state was achieved by curved space corrections to the energy–momentum tensor of a scalar field. The scalar field state employed in the original version of inflation is called a *false vacuum*, since the state temporarily acts as if it were the state of lowest possible energy density. Classically, this state would be completely stable, because there would be no energy available to allow the scalar field to cross the potential energy barrier that separates it from states of lower energy. Quantum mechanically, however, the state would decay by tunneling [3]. Initially, it was hoped that this tunneling process could successfully end inflation, but it was soon found that the randomness of false vacuum decay would produce catastrophically large inhomogeneities. These problems were summarized in Ref. [1], and described more fully by Hawking et al. [4] and by Guth and Weinberg [5].

This “graceful exit” problem was solved by the invention of the new inflationary universe model by Linde [6] and by Albrecht and Steinhardt [7]. New inflation achieved all the successes that had been hoped for in the context of the original version. In this theory inflation is driven by a scalar field perched on a plateau of the potential energy diagram, as shown in Fig. 1. Such a scalar field is generically called the *inflaton*. If the plateau is flat enough, such a state can be stable enough for successful inflation. Soon afterwards Linde showed that the inflaton potential need not have either a local minimum or a gentle plateau: in the scenario he dubbed *chaotic inflation* [8], the inflaton potential can be as simple as

$$V(\phi) = \frac{1}{2}m^2\phi^2, \quad (1)$$

provided that ϕ begins at a large enough value so that inflation can occur as it relaxes. For simplicity of language, I will stretch the meaning of the phrase “false vacuum” to include all of these cases; that is, I will use the phrase to denote any state with a high-energy density that cannot be rapidly decreased. Note that while inflation was originally developed in the context of grand unified theories, the only real requirement on the particle physics is the existence of a false vacuum state.

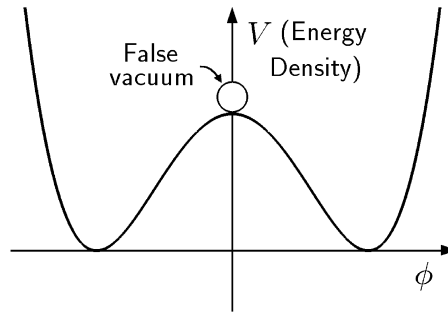


Fig. 1. Generic form of the potential for the new inflationary scenario.

2.1. The new inflationary scenario

In this section I will summarize the workings of new inflation, and in the following section I will discuss chaotic inflation. While more complicated possibilities (e.g. hybrid inflation [9–13] and supernatural inflation [14]) appear very plausible, the basic scenarios of new and chaotic inflation will be sufficient to illustrate the physical effects that I want to discuss in this article.

Suppose that the energy density of a state is approximately equal to a constant value ρ_f . Then, if a region filled with this state of matter expanded by an amount dV , its energy would have to increase by

$$dU = \rho_f dV . \quad (2)$$

This energy must be supplied by whatever force is causing the expansion, which means that the force must be pulling against a negative pressure. The work done by the force is given by

$$dW = - p_f dV , \quad (3)$$

where p_f is the pressure inside the expanding region. Equating the work with the change in energy, one finds

$$p_f = - \rho_f . \quad (4)$$

This negative pressure is the driving force behind inflation. When one puts this negative pressure into Einstein's equations, one finds that it leads to a repulsion, causing such a region to undergo exponential expansion. If the region can be approximated as isotropic and homogeneous, this result can be seen from the standard Friedmann–Robertson–Walker (FRW) equations:

$$d^2a/dt^2 = - (4\pi/3)G(\rho + 3p)a = (8\pi/3)G\rho_f a , \quad (5)$$

where $a(t)$ is the scale factor, G is Newton's constant, and we adopt units for which $\hbar = c = 1$. For late times the growing solution to this equation has the form

$$a(t) \propto e^{\chi t} \quad \text{where } \chi = \sqrt{(8\pi/3) G\rho_f} . \quad (6)$$

Of course, inflationary theorists prefer not to assume that the universe began homogeneously and isotropically, but there is considerable evidence for the “cosmological no-hair conjecture” [15], which implies that a wide class of initial states will approach this exponentially expanding solution.

The basic scenario of new inflation begins by assuming that at least some patch of the early universe was in this peculiar false vacuum state. In the original papers [6,7] this initial condition was motivated by the fact that, in many quantum field theories, the false vacuum resulted naturally from the supercooling of an initially hot state in thermal equilibrium. It was soon found, however, that quantum fluctuations in the rolling inflaton field give rise to density perturbations in the universe [16–20], and that these density perturbations would be much larger than observed unless the inflaton field is very weakly coupled. For such weak coupling there would be no time for an initially nonthermal state to reach thermal equilibrium. Nonetheless, since thermal equilibrium describes a probability distribution in which all states of a given energy are weighted equally, the fact that thermal equilibrium leads to a false vacuum implies that there are many ways of reaching a false vacuum. Thus, even in the absence of thermal equilibrium – even if the universe started in a highly chaotic initial state – it seems reasonable to assume that some small patches of the early universe settled into the false vacuum state, as was suggested for example in Ref. [21]. Linde [8] pointed out that even highly improbable initial patches could be important if they inflated, since the exponential expansion could still cause such patches to dominate the volume of the universe. One might hope ultimately to calculate the probability of regions settling into the false vacuum from a quantum description of cosmogenesis, but I will argue in Section 5 that this probability is quite irrelevant in the context of eternal inflation.

Once a region of false vacuum materializes, the physics of the subsequent evolution is rather straightforward. The gravitational repulsion caused by the negative pressure will drive the region into a period of exponential expansion. If the energy density of the false vacuum is at the grand unified theory scale ($\rho_f \approx (2 \times 10^{16} \text{ GeV})^4$), Eq. (6) shows that the time constant χ^{-1} of the exponential expansion would be about 10^{-38} s. For inflation to achieve its goals, this patch has to expand exponentially for at least 60 e-foldings. Then, because the false vacuum is only metastable (the inflaton field is perched on top of the hill of the potential energy diagram of Fig. 1), eventually it will decay. The inflaton field will roll off the hill, ending inflation. When it does, the energy density that has been locked in the inflaton field is released. Because of the coupling of the inflaton to other fields, that energy becomes thermalized to produce a hot soup of particles, which is exactly what had always been taken as the starting point of the standard big bang theory before inflation was introduced. From here on the scenario joins the standard big bang description. The role of inflation is to establish dynamically the initial conditions which otherwise have to be postulated.

The inflationary mechanism produces an entire universe starting from essentially nothing, so one needs to answer the question of where the energy of the universe comes from. The answer is that it comes from the gravitational field. The universe did not begin with this colossal energy stored in the gravitational field, but rather the gravitational field can supply the energy because its energy can become negative without bound. As more and more positive energy materializes in the form of an ever-growing region filled with a high-energy scalar field, more and more negative energy materializes in the form of an expanding region filled with a gravitational field. The total energy remains constant at some very small value, and could in fact be exactly zero. There is nothing

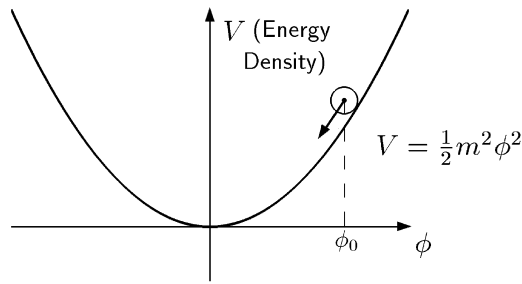


Fig. 2. Generic form of the potential for the chaotic inflationary scenario.

known that places any limit on the amount of inflation that can occur while the total energy remains exactly zero.¹

2.2. Chaotic inflation

Chaotic inflation [8] can occur in the context of a more general class of potential energy functions. In particular, even a potential energy function as simple as Eq. (1) – describing a scalar field with a mass and no interaction – is sufficient to describe chaotic inflation. Chaotic inflation is illustrated in Fig. 2. In this case there is no state that bears any obvious resemblance to the false vacuum of new inflation. Instead the scenario works by supposing that chaotic conditions in the early universe produced one or more patches in which the inflaton field ϕ was at some high value $\phi = \phi_0$ on the potential energy curve. Inflation occurs as the inflaton field rolls down the hill. As long as the initial value ϕ_0 is sufficiently large, there will be sufficient inflation to solve all the problems that inflation is intended to solve.

The equations describing chaotic inflation can be written simply, provided that we assume that the universe is already flat enough so that we do not need to include a curvature term. The field equation for the inflaton field in the expanding universe is

$$\ddot{\phi} + 3H\dot{\phi} = -\partial V/\partial\phi, \quad (7)$$

where the overdot denotes a derivative with respect to time t , and H is the time-dependent Hubble parameter given by

$$H^2 = (8\pi/3)GV. \quad (8)$$

For the toy-model potential energy of Eq. (1), these equations have a very simple solution:

$$\phi = \phi_0 - (m/\sqrt{12\pi G})t. \quad (9)$$

¹ In Newtonian mechanics the energy density of a gravitational field is unambiguously negative; it can be derived by the same methods used for the Coulomb field, but the force law has the opposite sign. In general relativity there is no coordinate-invariant way of expressing the energy in a space that is not asymptotically flat, so many experts prefer to say that the total energy is undefined. Either way, there is agreement that inflation is consistent with the general relativistic description of energy conservation.

One can then calculate the number N of inflationary e-foldings, which is given by

$$N = \int_{\phi=\phi_0}^{\phi=0} H(t) dt = 2\pi G \phi_0^2 . \quad (10)$$

In this toy model N depends only on ϕ_0 and not on the inflaton mass m . Thus, the number of e-foldings will exceed 60 provided that

$$\phi_0 > \sqrt{(60/2\pi)M_{\text{P}}} \approx 3.1M_{\text{P}} , \quad (11)$$

where $M_{\text{P}} \equiv 1/\sqrt{G} = 1.22 \times 10^{19}$ GeV is the Planck mass. Although this is a super-Planckian value for the scalar field, the energy density need not be super-Planckian:

$$\rho_0 = \frac{1}{2}m^2\phi_0^2 > (60/4\pi)M_{\text{P}}^2m^2 . \quad (12)$$

For example, if $m = 10^{16}$ GeV, then the potential energy density is only $3 \times 10^{-6} M_{\text{P}}^4$. Since it is presumably the energy density and not the value of the field that is relevant to gravity, it seems reasonable to assume that the chaotic inflation scenario will not be dramatically affected by corrections from quantum gravity.

3. Evidence for inflation

No matter which form of inflation we might envision, we would like to know what is the evidence that our universe underwent a period of inflation. The answer is pretty much the same no matter which form of inflation we are discussing. In my opinion, the evidence that our universe is the result of some form of inflation is very solid. Since the term *inflation* encompasses a wide range of detailed theories, it is hard to imagine any reasonable alternative. The basic arguments are as follows:

(1) *The universe is big*: First of all, we know that the universe is incredibly large: the visible part of the universe contains about 10^{90} particles. Since we have all grown up in a large universe, it is easy to take this fact for granted: of course the universe is big, it is the whole universe! In “standard” FRW cosmology, without inflation, one simply postulates that about 10^{90} or more particles were here from the start. However, in the context of present-day cosmology, many of us hope that even the creation of the universe can be described in scientific terms. Thus, we are led to at least think about a theory that might explain how the universe got to be so big. Whatever that theory is, it has to somehow explain the number of particles, 10^{90} or more. However, it is hard to imagine such a number arising from a calculation in which the input consists only of geometrical quantities, quantities associated with simple dynamics, and factors of 2 or π . The easiest way by far to get a huge number, with only modest numbers as input, is for the calculation to involve an exponential. The exponential expansion of inflation reduces the problem of explaining 10^{90} particles to the problem of explaining 60 or 70 e-foldings of inflation. In fact, it is easy to construct underlying particle theories that will give far more than 70 e-foldings of inflation. Inflationary cosmology therefore suggests that, even though the observed universe is incredibly large, it is only an infinitesimal fraction of the entire universe.

(2) *The Hubble expansion*: The Hubble expansion is also easy to take for granted, since we have all known about it from our earliest readings in cosmology. In standard FRW cosmology, the Hubble

expansion is part of the list of postulates that define the initial conditions. But inflation actually offers the possibility of explaining how the Hubble expansion began. The repulsive gravity associated with the false vacuum is just what Hubble ordered. It is exactly the kind of force needed to propel the universe into a pattern of motion in which each pair of particles is moving apart with a velocity proportional to their separation.

(3) *Homogeneity and isotropy*: The degree of uniformity in the universe is startling. The intensity of the cosmic background radiation is the same in all directions, after it is corrected for the motion of the Earth, to the incredible precision of one part in 100,000. To get some feeling for how high this precision is, we can imagine a marble that is spherical to one part in 100,000. The surface of the marble would have to be shaped to an accuracy of about 1000 Å, a quarter of the wavelength of light.

Although modern technology makes it possible to grind lenses to quarter-wavelength accuracy, we would nonetheless be shocked if we unearthed a stone, produced by natural processes, that was round to an accuracy of 1000 Å. If we try to imagine that such a stone were found, I am sure that no one would accept an explanation of its origin which simply proposed that the stone started out perfectly round. Similarly, I do not think it makes sense to consider any theory of cosmogenesis that cannot offer some explanation of how the universe became so incredibly isotropic.

The cosmic background radiation was released about 300,000 years after the big bang, after the universe cooled enough so that the opaque plasma neutralized into a transparent gas. The cosmic background radiation photons have mostly been traveling on straight lines since then, so they provide an image of what the universe looked like at 300,000 years after the big bang. The observed uniformity of the radiation therefore implies that the observed universe had become uniform in temperature by that time. In standard FRW cosmology, a simple calculation shows that the uniformity could be established so quickly only if signals could propagate at 100 times the speed of light, a proposition clearly contradicting the known laws of physics. In inflationary cosmology, however, the uniformity is easily explained. The uniformity is created initially on microscopic scales, by normal thermal-equilibrium processes, and then inflation takes over and stretches the regions of uniformity to become large enough to encompass the observed universe.

(4) *The flatness problem*: I find the flatness problem particularly impressive, because of the extraordinary numbers that it involves. The problem concerns the value of the ratio

$$\Omega_{\text{tot}} \equiv \rho_{\text{tot}}/\rho_c \text{ ,} \quad (13)$$

where ρ_{tot} is the average total mass density of the universe and $\rho_c = 3H^2/8\pi G$ is the critical density, the density that would make the universe spatially flat. (In the definition of “total mass density”, I am including the vacuum energy $\rho_{\text{vac}} = \Lambda/8\pi G$ associated with the cosmological constant Λ , if it is nonzero.)

There is general agreement that the present value of Ω_{tot} satisfies

$$0.1 \lesssim \Omega_0 \lesssim 2 \text{ ,} \quad (14)$$

but it is hard to pinpoint the value with more precision. Despite the breadth of this range, the value of Ω at early times is highly constrained, since $\Omega = 1$ is an unstable equilibrium point of the standard model evolution. Thus, if Ω was ever *exactly* equal to one, it would remain exactly one forever. However, if Ω differed slightly from one in the early universe, that difference – whether positive or negative – would be amplified with time. In particular, it can be shown that $\Omega - 1$ grows

as

$$\Omega - 1 \propto \begin{cases} t & \text{(during the radiation-dominated era) ,} \\ t^{2/3} & \text{(during the matter-dominated era) .} \end{cases} \quad (15)$$

At $t = 1$ s, for example, when the processes of big bang nucleosynthesis were just beginning, Dicke and Peebles [22] pointed out that Ω must have equaled one to an accuracy of one part in 10^{15} . Classical cosmology provides no explanation for this fact – it is simply assumed as part of the initial conditions. In the context of modern particle theory, where we try to push things all the way back to the Planck time, 10^{-43} s, the problem becomes even more extreme. If one specifies the value of Ω at the Planck time, it has to equal one to 58 decimal places in order to be anywhere in the allowed range today.

While this extraordinary flatness of the early universe has no explanation in classical FRW cosmology, it is a natural prediction for inflationary cosmology. During the inflationary period, instead of Ω being driven away from one as described by Eq. (15), Ω is driven towards one, with exponential swiftness:

$$\Omega - 1 \propto e^{-2H_{\text{inf}}t} , \quad (16)$$

where H_{inf} is the Hubble parameter during inflation. Thus, as long as there is a long enough period of inflation, Ω can start at almost any value, and it will be driven to unity by the exponential expansion.

(5) *Absence of magnetic monopoles:* All grand unified theories predict that there should be, in the spectrum of possible particles, extremely massive particles carrying a net magnetic charge. By combining grand unified theories with classical cosmology without inflation, Preskill [23] found that magnetic monopoles would be produced so copiously that they would outweigh everything else in the universe by a factor of about 10^{12} . A mass density this large would cause the inferred age of the universe to drop to about 30,000 years! Inflation is certainly the simplest known mechanism to eliminate monopoles from the visible universe, even though they are still in the spectrum of possible particles. The monopoles are eliminated simply by arranging the parameters so that inflation takes place after (or during) monopole production, so the monopole density is diluted to a completely negligible level.

(6) *Anisotropy of the cosmic background radiation:* The process of inflation smooths the universe essentially completely, but density fluctuations are generated as inflation ends by the quantum fluctuations of the inflaton field. Generically, these are adiabatic Gaussian fluctuations with a nearly scale-invariant spectrum [16–20]. New data is arriving quickly, but so far the observations are in excellent agreement with the predictions of the simplest inflationary models. For a review, see for example [24], who find that the combined data give a slope of the primordial power spectrum within 5% of the preferred scale-invariant value.

4. Eternal inflation: mechanisms

The remainder of this article will discuss eternal inflation – the questions that it can answer, and the questions that it raises. In this section I discuss the mechanisms that make eternal inflation

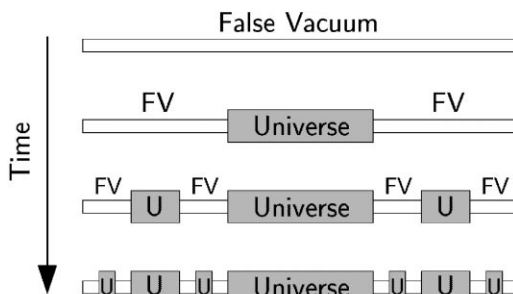


Fig. 3. A schematic illustration of eternal inflation.

possible, leaving the other issues for the following sections. I will discuss eternal inflation first in the context of new inflation, and then in the context of chaotic inflation, where it is more subtle.

4.1. *Eternal new inflation*

The eternal nature of new inflation was first discovered by Steinhardt [25] and Vilenkin [26] in 1983. Although the false vacuum is a metastable state, the decay of the false vacuum is an exponential process, very much like the decay of any radioactive or unstable substance. The probability of finding the inflaton field at the top of the plateau in its potential energy diagram does not fall sharply to zero, but instead trails off exponentially with time [27]. However, unlike a normal radioactive substance, the false vacuum exponentially expands at the same time that it decays. In fact, in any successful inflationary model the rate of exponential expansion is always much faster than the rate of exponential decay. Therefore, even though the false vacuum is decaying, it never disappears, and in fact the total volume of the false vacuum, once inflation starts, continues to grow exponentially with time, ad infinitum.

Fig. 3 shows a schematic diagram of an eternally inflating universe. The top bar indicates a region of false vacuum. The evolution of this region is shown by the successive bars moving downward, except that the expansion could not be shown while still fitting all the bars on the page. So the region is shown as having a fixed size in comoving coordinates, while the scale factor, which is not shown, increases from each bar to the next. As a concrete example, suppose that the scale factor for each bar is three times larger than for the previous bar. If we follow the region of false vacuum as it evolves from the situation shown in the top bar to the situation shown in the second bar, in about one-third of the region the scalar field rolls down the hill of the potential energy diagram, precipitating a local big bang that will evolve into something that will eventually appear to its inhabitants as a universe. This local big bang region is shown in gray and labelled “universe”. Meanwhile, however, the space has expanded so much that each of the two remaining regions of false vacuum is the same size as the starting region. Thus, if we follow the region for another time interval of the same duration, each of these regions of false vacuum will break up, with about one-third of each evolving into a local universe, as shown on the third bar from the top. Now there are four remaining regions of false vacuum, and again each is as large as the starting region. This process will repeat itself literally forever, producing a kind of a fractal structure to the universe,

resulting in an infinite number of the local universes shown in gray. There is no standard name for these local universes, but they are often called bubble universes. I prefer, however, to call them pocket universes, to avoid the suggestion that they are round. While bubbles formed in first-order phase transitions are round [28], the local universes formed in eternal new inflation are generally very irregular, as can be seen, for example, in the two-dimensional simulation by Vanchurin, Vilenkin, and Winitzki in Fig. 2 of Ref. [29].

The diagram in Fig. 3 is of course an idealization. The real universe is three dimensional, while the diagram illustrates a schematic one-dimensional universe. It is also important that the decay of the false vacuum is really a random process, while the diagram was constructed to show a very systematic decay, because it is easier to draw and to think about. When these inaccuracies are corrected, we are still left with a scenario in which inflation leads asymptotically to a fractal structure [30] in which the universe as a whole is populated by pocket universes on arbitrarily small comoving scales. Of course, this fractal structure is entirely on distance scales much too large to be observed, so we cannot expect astronomers to see it. Nonetheless, one does have to think about the fractal structure if one wants to understand the very large-scale structure of the spacetime produced by inflation.

Most important of all is the simple statement that once inflation happens, it produces not just one universe, but an infinite number of universes.

4.2. Eternal chaotic inflation

The eternal nature of new inflation depends crucially on the scalar field lingering at the top of the plateau of Fig. 1. Since the potential function for chaotic inflation, Fig. 2, does not have a plateau, it is not obvious how eternal inflation can happen in this context. Nonetheless, Andrei Linde [31] showed in 1986 that chaotic inflation can also be eternal.

The important point is that quantum fluctuations play an important role in all inflationary models. Quantum fluctuations are invariably important on very small scales, and with inflation these very small scales are rapidly stretched to become macroscopic and even astronomical. Thus, the scalar field associated with inflation has very evident quantum effects.

When the mass of the scalar field is small compared to the Hubble parameter H , these quantum effects are accurately summarized by saying that the quantum fluctuations cause the field to undergo a random walk. It is useful to divide space into regions of physical size H^{-1} , and to discuss the average value of the scalar field ϕ within a given region. In a time H^{-1} , the effect of the quantum fluctuations is equivalent to a random Gaussian jump of zero mean and a root-mean-squared magnitude [16,32–34] given by

$$\Delta\phi_{\text{qu}} = H/2\pi . \quad (17)$$

This random quantum jump is superimposed on the classical motion, as indicated in Fig. 4.

To illustrate how eternal inflation happens in the simplest context, let us consider again the free scalar field described by the potential function of Eq. (1). We consider a region of physical radius H^{-1} , in which the field has an average value ϕ . Using Eq. (9) along with Eqs. (8) and (1), one finds that the magnitude of the classical change that the field will undergo in a time H^{-1} is given in by

$$\Delta\phi_{\text{cl}} = (M_{\text{P}}m/\sqrt{12\pi})H^{-1} = (1/4\pi)M_{\text{P}}^2/\phi . \quad (18)$$

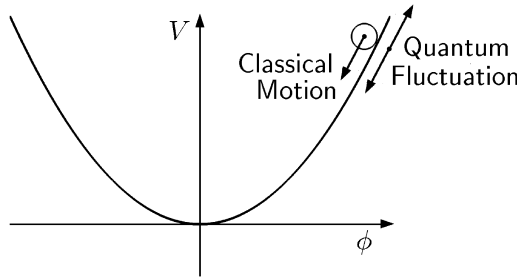


Fig. 4. Evolution of the inflaton field during eternal chaotic inflation.

Let ϕ^* denote the value of ϕ which is sufficiently large so that

$$\Delta\phi_{\text{qu}}(\phi^*) = \Delta\phi_{\text{cl}}(\phi^*) , \tag{19}$$

which can easily be solved to find

$$\phi^* = (3/16\pi)^{1/4} M_{\text{P}}^{3/2}/m^{1/2} . \tag{20}$$

Now consider what happens to the region if its initial average value of ϕ is equal to ϕ^* . In a time interval H^{-1} , the volume of the region will increase by $e^3 \approx 20$. At the end of the time interval we can divide the original region into 20 regions of the same volume as the original, and in each region the average scalar field can be written as

$$\phi = \phi^* + \Delta\phi_{\text{cl}} + \delta\phi , \tag{21}$$

where $\delta\phi$ denotes the random quantum jump, which is drawn from a Gaussian probability distribution with standard deviation $\Delta\phi_{\text{qu}} = \Delta\phi_{\text{cl}}$. Gaussian statistics imply that there is a 15.9% chance that a Gaussian random variable will exceed its mean by more than one standard deviation, and therefore there is a 15.9% chance that the net change in ϕ will be positive. Since there are now 20 regions of the original volume, on average the value of ϕ will exceed the original value in 3.2 of these regions. Thus the volume for which $\phi \geq \phi^*$ does not (on average) decrease, but instead increases by more than a factor of 3. Since this argument can be iterated, the expectation value of the volume for which $\phi \geq \phi^*$ increases exponentially with time. Typically, therefore, inflation never ends, but instead the volume of the inflating region grows exponentially without bound. The minimum field value for eternal inflation is slightly below ϕ^* , since a volume increase by a factor of 3.2 is more than necessary – any factor greater than one would be sufficient. A short calculation shows that the minimal value for eternal inflation is $0.78\phi^*$.

While the value of ϕ^* is larger than M_{P} , it is important to note that the energy density can still be much smaller than Planck scale:

$$V(\phi^*) = \frac{1}{2}m^2\phi^{*2} = \sqrt{(3/64\pi)m}M_{\text{P}}^3 , \tag{22}$$

which for $m = 10^{16}$ GeV gives an energy density of $1 \times 10^{-4}M_{\text{P}}^4$.

If one repeats the argument with a potential function

$$V(\phi) = \frac{1}{4}\lambda\phi^4 , \tag{23}$$

one finds [35] that

$$\phi^* = (3/2\pi\lambda)^{1/6} M_{\text{P}} , \quad (24)$$

$$V(\phi^*) = (3/16\pi)^{2/3} \lambda^{1/3} M_{\text{P}}^4 . \quad (25)$$

Since one requires λ to be very small in any case so that density perturbations are not too large, one finds again that eternal inflation is predicted to happen at an energy density well below the Planck scale.

5. Eternal inflation: implications

In spite of the fact that the other universes created by eternal inflation are too remote to imagine observing directly, I nonetheless claim that eternal inflation has real consequences in terms of the way we extract predictions from theoretical models. Specifically, there are three consequences of eternal inflation that I will discuss.

First, eternal inflation implies that all hypotheses about the ultimate initial conditions for the universe – such as the Hartle–Hawking [36] no boundary proposal, the tunneling proposals by Vilenkin [37] or Linde [38], or the more recent Hawking–Turok instanton [39] – become totally divorced from observation. That is, one would expect that if inflation is to continue arbitrarily far into the future with the production of an infinite number of pocket universes, then the statistical properties of the inflating region should approach a steady state which is independent of the initial conditions. Unfortunately, attempts to quantitatively study this steady state are severely limited by several factors. First, there are ambiguities in defining probabilities, which will be discussed later. In addition, the steady-state properties seem to depend strongly on super-Planckian physics which we do not understand. That is, the same quantum fluctuations that make eternal chaotic inflation possible tend to drive the scalar field further and further up the potential energy curve, so attempts to quantify the steady-state probability distribution [40,41] require the imposition of some kind of a boundary condition at large ϕ . Although these problems remain unsolved, I still believe that it is reasonable to assume that in the course of its unending evolution, an eternally inflating universe would lose all memory of the state in which it started.

Even if the universe forgets the details of its genesis, however, I would not assume that the question of how the universe began would lose its interest. While eternally inflating universes continue forever once they start, they are presumably not eternal into the past. (The word *eternal* is therefore not technically correct – it would be more precise to call this scenario *semi-eternal* or *future-eternal*.) While the issue is not completely settled, it appears likely that eternally inflating universes must necessarily have a beginning. Borde and Vilenkin [42] have shown, subject to various assumptions, that spacetimes that are future-eternal must have an initial singularity, in the sense that they cannot be past null geodesically complete. The proof, however, requires the weak energy condition, which can be violated by quantum fluctuations [43]. In any case, no one has constructed a viable model without a beginning, and certainly nothing that we know can rule out the possibility of a beginning. The possibility of a quantum origin of the universe is very attractive, and will no doubt be a subject of interest for some time. Eternal inflation, however, seems to imply that the entire study will have to be conducted with literally no input from observation.

A second consequence of eternal inflation is that the probability of the onset of inflation becomes totally irrelevant, provided that the probability is not identically zero. Various authors in the past have argued that one type of inflation is more plausible than another, because the initial conditions that it requires appear more likely to have occurred. In the context of eternal inflation, however, such arguments have no significance.

To illustrate the insignificance of the probability of the onset of inflation, I will use a numerical example. We will imagine comparing two different versions of inflation, which I will call Types A and B. They are both eternally inflating – but Type A will have a higher probability of starting, while Type B will be a little faster in its exponential expansion rate. Since I am trying to show that the higher starting probability of Type A is irrelevant, I will choose my numbers to be extremely generous to Type A. First, we must choose a number for how much more probable it is for Type A inflation to begin, relative to Type B. A googol, 10^{100} , is usually considered a large number – it is some 20 orders of magnitude larger than the total number of baryons in the visible universe. But I will be more generous: I will assume that Type A inflation is more likely to start than Type B inflation by a factor of $10^{1,000,000}$. Type B inflation, however, expands just a little bit faster, say by 0.001%. We need to choose a time constant for the exponential expansion, which I will take to be a typical grand unified theory scale, $\tau = 10^{-37}$ s. (τ represents the time constant for the overall expansion factor, which takes into account both the inflationary expansion and the exponential decay of the false vacuum.) Finally, we need to choose a length of time to let the system evolve. In principle, this time interval is infinite (the inflation is eternal into the future), but to be conservative we will follow the system for only 1 s.

We imagine starting a statistical ensemble of universes at $t = 0$, with an expectation value for the volume of Type A inflation exceeding that of Type B inflation by $10^{1,000,000}$. For brevity, I will use the term “weight” to refer to the ensemble expectation value of the volume. Thus, at $t = 0$ the weights of Type A inflation and Type B inflation will have the ratio

$$W_B/W_A|_{t=0} = 10^{-1,000,000} . \quad (26)$$

After 1 s of evolution, the expansion factors for Types A and B inflation will be

$$Z_A = e^{t/\tau} = e^{10^{37}} , \quad (27)$$

$$\begin{aligned} Z_B &= e^{1.00001 t/\tau} = e^{0.00001 t/\tau} Z_A \\ &= e^{10^{32}} Z_A \approx 10^{4.3 \times 10^{31}} Z_A . \end{aligned} \quad (28)$$

The weights at the end of one second are proportional to these expansion factors, so

$$W_B/W_A|_{t=1 \text{ sec}} = 10^{(4.3 \times 10^{31} - 1,000,000)} . \quad (29)$$

Thus, the initial ratio of $10^{1,000,000}$ is vastly superseded by the difference in exponential expansion factors. In fact, we would have to calculate the exponent of Eq. (29) to an accuracy of 25 significant figures to be able to barely detect the effect of the initial factor of $10^{1,000,000}$.

One might criticize the above argument for being naive, as the concept of time was invoked without any specification of how the equal-time hypersurfaces are to be defined. I do not know a decisive answer to this objection; as I will discuss later, there are unresolved questions concerning the calculation of probabilities in eternally inflating spacetimes. Nonetheless, given that there is

actually an infinity of time available, it seems reasonable to believe that the form of inflation that expands the fastest will always dominate over the slower forms by an infinite factor.

A corollary to this argument is that new inflation is not dead. While the initial conditions necessary for new inflation cannot be justified on the basis of thermal equilibrium, as proposed in the original papers [6,7], in the context of eternal inflation it is sufficient to conclude that the probability for the required initial conditions is nonzero. Since the resulting scenario does not depend on the words that are used to justify the initial state, the standard treatment of new inflation remains valid.

A third consequence of eternal inflation is the possibility that it offers to rescue the predictive power of theoretical physics. Here I have in mind the status of string theory, or the theory known as M theory, into which string theory has evolved. The theory itself has an elegant uniqueness, but nonetheless it is not at all clear that the theory possesses a unique vacuum. Since predictions will ultimately depend on the properties of the vacuum, the predictive power of string/M theory may be limited. Eternal inflation, however, provides a possible mechanism to remedy this problem. Even if many types of vacua are equally stable, it may turn out that one of them leads to a maximal rate of inflation. If so, then this type of vacuum will dominate the universe, even if its expansion rate is only infinitesimally larger than the other possibilities. Thus, eternal inflation might allow physicists to extract unique predictions, in spite of the multiplicity of stable vacua.

6. Difficulties in calculating probabilities

In an eternally inflating universe, anything that can happen will happen; in fact, it will happen an infinite number of times. Thus, the question of what is possible becomes trivial – anything is possible, unless it violates some absolute conservation law. To extract predictions from the theory, we must therefore learn to distinguish the probable from the improbable.

However, as soon as one attempts to define probabilities in an eternally inflating spacetime, one discovers ambiguities. The problem is that the sample space is infinite, in that an eternally inflating universe produces an infinite number of pocket universes. The fraction of universes with any particular property is therefore equal to infinity divided by infinity – a meaningless ratio. To obtain a well-defined answer, one needs to invoke some method of regularization.

To understand the nature of the problem, it is useful to think about the integers as a model system with an infinite number of entities. We can ask, for example, what fraction of the integers are odd. Most people would presumably say that the answer is $\frac{1}{2}$, since the integers alternate between odd and even. That is, if the string of integers is truncated after the N th, then the fraction of odd integers in the string is exactly $\frac{1}{2}$ if N is even, and is $(N + 1)/2N$ if N is odd. In any case, the fraction approaches $\frac{1}{2}$ as N approaches infinity.

However, the ambiguity of the answer can be seen if one imagines other orderings for the integers. One could, if one wished, order the integers as

$$1, 3, 2, 5, 7, 4, 9, 11, 6, \dots, \quad (30)$$

always writing two odd integers followed by one even integer. This series includes each integer exactly once, just like the usual sequence $(1, 2, 3, 4, \dots)$. The integers are just arranged in an unusual order. However, if we truncate the sequence shown in Eq. (30) after the N th entry, and then take the

limit $N \rightarrow \infty$, we would conclude that $\frac{2}{3}$ of the integers are odd. Thus, we find that the definition of probability on an infinite set requires some method of truncation, and that the answer can depend nontrivially on the method that is used.

In the case of eternally inflating spacetimes, the natural choice of truncation might be to order the pocket universes in the sequence in which they form. However, we must remember that each pocket universe fills its own future light cone, so no pocket universe forms in the future light cone of another. Any two pocket universes are space-like separated from each other, so some observers will see one as forming first, while other observers will see the opposite. One can arbitrarily choose equal-time surfaces that foliate the spacetime, and then truncate at some value of t , but this recipe is not unique. In practice, different ways of choosing equal-time surfaces give different results.

7. The youngness paradox

If one chooses a truncation in the most naive way, one is led to a set of very peculiar results which I call the *youngness paradox*.

Specifically, suppose that one constructs a Robertson–Walker coordinate system while the model universe is still in the false vacuum (de Sitter) phase, before any pocket universes have formed. One can then propagate this coordinate system forward with a synchronous gauge condition,² and one can define probabilities by truncating at a fixed value t_f of the synchronous time coordinate t . That is, the probability of any particular property can be taken to be proportional to the volume on the $t = t_f$ hypersurface which has that property. This method of defining probabilities was studied in detail by Linde, Linde, and Mezhlumian, in a paper with the memorable title “Do we live in the center of the world?” [44]. I will refer to probabilities defined in this way as synchronous gauge probabilities.

The youngness paradox is caused by the fact that the volume of false vacuum is growing exponentially with time with an extraordinary time constant, in the vicinity of 10^{-37} s. Since the rate at which pocket universes form is proportional to the volume of false vacuum, this rate is increasing exponentially with the same time constant. That means that in each second the number of pocket universes that exist is multiplied by a factor of $\exp\{10^{37}\}$. At any given time, therefore, almost all of the pocket universes that exist are universes that formed very very recently, within the last several time constants. The population of pocket universes is therefore an incredibly youth-dominated society, in which the mature universes are vastly outnumbered by universes that have just barely begun to evolve. Although the mature universes have a larger volume, this multiplicative factor is of little importance, since in synchronous coordinates the volume no longer grows exponentially once the pocket universe forms.

Probability calculations in this youth-dominated ensemble lead to peculiar results, as discussed in Ref. [44]. These authors considered the expected behavior of the mass density in our vicinity, concluding that we should find ourselves very near the center of a spherical low-density region. Here I would like to discuss a less physical but simpler question, just to illustrate the paradoxes

² By a synchronous gauge condition, I mean that each equal-time hypersurface is obtained by propagating every point on the previous hypersurface by a fixed infinitesimal time interval Δt in the direction normal to the hypersurface.

associated with synchronous gauge probabilities. Specifically, I will consider the question: “Are there any other civilizations in the visible universe that are more advanced than ours?”. Intuitively I would not expect inflation to make any predictions about this question, but I will argue that the synchronous gauge probability distribution strongly implies that there is no civilization in the visible universe more advanced than we are.

Suppose that we have reached some level of advancement, and suppose that t_{\min} represents the minimum amount of time needed for a civilization as advanced as we are to evolve, starting from the moment of the decay of the false vacuum – the start of the big bang. The reader might object on the grounds that there are many possible measures of advancement, but I would respond by inviting the reader to pick any measure she chooses; the argument that I am about to give should apply to all of them. The reader might alternatively claim that there is no sharp minimum t_{\min} , but instead we should describe the problem in terms of a function which gives the probability that, for any given pocket universe, a civilization as advanced as we are would develop by time t . I believe, however, that the introduction of such a probability distribution would merely complicate the argument, without changing the result. So, for simplicity of discussion, I will assume that there is some sharply defined minimum time t_{\min} required for a civilization as advanced as ours to develop.

Since we exist, our pocket universe must have an age t_0 satisfying

$$t_0 \geq t_{\min} . \quad (31)$$

Suppose, however, that there is some civilization in our pocket universe that is more advanced than we are, let us say by 1 s. In that case Eq. (31) is not sufficient, but instead the age of our pocket universe would have to satisfy

$$t_0 \geq t_{\min} + 1 \text{ s} . \quad (32)$$

However, in the synchronous gauge probability distribution, universes that satisfy Eq. (32) are outnumbered by universes that satisfy Eq. (31) by a factor of approximately $\exp\{10^{37}\}$. Thus, if we know only that we are living in a pocket universe that satisfies Eq. (31), it is extremely improbable that it also satisfies Eq. (32). We would conclude, therefore, that it is extraordinarily improbable that there is a civilization in our pocket universe that is at least 1 s more advanced than we are.

Perhaps this argument explains why SETI has not found any signals from alien civilizations, but I find it more plausible that it is merely a symptom that the synchronous gauge probability distribution is not the right one.

8. An alternative probability prescription

Since the probability measure depends on the method used to truncate the infinite spacetime of eternal inflation, we are not forced to accept the consequences of the synchronous gauge probabilities. A very attractive alternative has been proposed by Vilenkin [45], and developed further by Vanchurin et al. [29].

The key idea of the Vilenkin proposal is to define probabilities within a single-pocket universe (which he describes more precisely as a connected, thermalized domain). Thus, unlike the synchronous gauge method, there is no comparison between old pocket universes and young ones. To

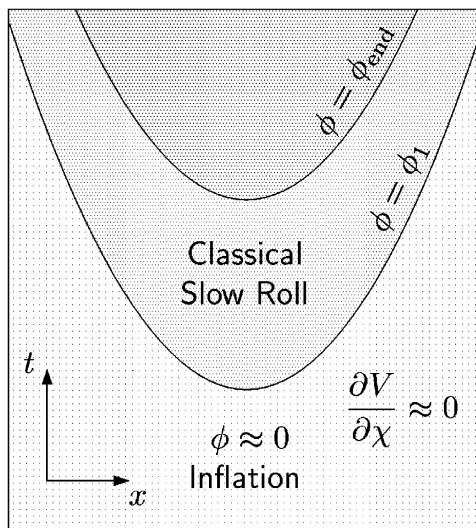


Fig. 5. A schematic picture of a pocket universe, illustrating Vilenkin's proposal for the calculation of probabilities.

justify this approach it is crucial to recognize that each pocket universe is infinite, even if one starts the model with a finite region of de Sitter space. The infinite volume arises in the same way as it does for the special case of Coleman–de Luccia bubbles [28], the interior of which are open Robertson–Walker universes. From the outside one often describes such bubbles in a coordinate system in which they are finite at any fixed time, but in which they grow without bound. On the inside, however, the natural coordinate system is the one that reflects the intrinsic homogeneity, in which the space is infinite at any given time. The infinity of time, as seen from the outside, becomes an infinity of spatial extent as seen on the inside. Thus, at least for continuously variable parameters, a single-pocket universe provides an infinite sample space which can be used to define probabilities. The second key idea of Vilenkin's method is to use the inflaton field itself as the time variable, rather than the synchronous time variable discussed in the previous section.

This approach can be used, for example, to discuss the probability distribution for Ω in open inflationary models, or to discuss the probability distribution for some arbitrary field that has a flat potential energy function. If, however, the vacuum has a discrete parameter which is homogeneous within each pocket universe, but which takes on different values in different pocket universes, then this method does not apply.

The proposal can be described in terms of Fig. 5. We suppose that the theory includes an inflaton field ϕ of the new inflation type, and some set of fields χ_i which have flat potentials. The goal is to find the probability distribution for the fields χ_i . We assume that the evolution of the inflaton ϕ can be divided into three regimes, as shown on the figure. $\phi < \phi_1$ describes the eternally inflating regime, in which the evolution is governed by quantum diffusion. For $\phi_1 < \phi < \phi_{\text{end}}$, the evolution is described classically in a slow-roll approximation, so that $\dot{\phi} \equiv d\phi/dt$ can be expressed as a function of ϕ . For $\phi > \phi_{\text{end}}$ inflation is over, and the ϕ field no longer plays an important role in the evolution. The χ_i fields are assumed to have a finite range of values, such as angular variables, so that a flat probability distribution is normalizable. They are assumed to have a flat potential

energy function for $\phi > \phi_{\text{end}}$, so that they could settle at any value. They are also assumed to have a flat potential energy function for $\phi < \phi_1$, although they might interact with ϕ during the slow-roll regime, however, so that they can affect the rate of inflation.

Since the potential for the χ_i is flat for $\phi < \phi_1$, we can assume that they begin with a flat probability distribution $P_0(\chi_i) \equiv P(\chi_i, \phi_1)$ on the $\phi = \phi_1$ hypersurface. If the kinetic energy function for the χ_i is of the standard form, we take $P_0(\chi_i) = \text{const}$. If, however, the kinetic energy is nonstandard,

$$\mathcal{L}_{\text{kinetic}} = g^{ij}(\chi) \partial_\mu \chi_i \partial^\mu \chi_j \quad (33)$$

as is plausible for a field described in angular variables, then the initial probability distribution is assumed to take the reparameterization-invariant form

$$P_0(\chi_i) \propto \sqrt{\det g} . \quad (34)$$

During the slow-roll era, it is assumed that the χ_i fields evolve classically, so one can calculate the number of e-folds of inflation $N(\chi_i)$ as a function of the final value of the χ_i (i.e., the value of χ_i on the $\phi = \phi_{\text{end}}$ hypersurface). One can also calculate the final values χ_i in terms of the initial values χ_i^0 (i.e., the value of χ_i on the $\phi = \phi_1$ hypersurface). One then assumes that the probability density is enhanced by the volume inflation factor $e^{3N(\chi_i)}$, and that the evolution from χ_i^0 to χ_i results in a Jacobian factor. The (unnormalized) final probability distribution is thus given by

$$P(\chi_i, \phi_{\text{end}}) = P_0(\chi_i^0) e^{3N(\chi_i)} \det \partial \chi_j^0 / \partial \chi_k . \quad (35)$$

Alternatively, if the evolution of the χ_i during the slow-roll era is subject to quantum fluctuations, Ref. [29] shows how to write a Fokker–Planck equation which is equivalent to averaging the result of Eq. (35) over a collection of paths that result from interactions with a noise term.

The Vilenkin proposal sidesteps the youngness paradox by defining probabilities by the comparison of volumes within one-pocket universe. The youngness paradox, in contrast, arose when one considered a probability ensemble of all pocket universes at a fixed value of the synchronous gauge time coordinate – an ensemble that is overwhelmingly dominated by very young pocket universes.

The proposal has the drawback, however, that it cannot be used to compare the probabilities of discretely different alternatives. Furthermore, although the results of this method seem reasonable, I do not at this point find them compelling. That is, it is not clear what principles of physics or probability theory ensure that this particular method of regularizing the spacetime is the one that leads to correct predictions. Perhaps there is no way to answer this question, so we may be forced to accept this proposal, or something similar to it, as a postulate.

9. Conclusion

In this paper I have summarized the workings of inflation, and the arguments that strongly suggest that our universe is the product of inflation. I argued that inflation can explain the size, the Hubble expansion, the homogeneity, the isotropy, and the flatness of our universe, as well as the absence of magnetic monopoles, and even the characteristics of the nonuniformities. The detailed observations of the cosmic background radiation anisotropies continue to fall in line with

inflationary expectations, and the evidence for an accelerating universe fits well with the inflationary preference for a flat universe.

Next, I turned to the question of eternal inflation, claiming that essentially all inflationary models are eternal. In my opinion this makes inflation very robust: if it starts anywhere, at any time in all of eternity, it produces an infinite number of pocket universes. Eternal inflation has the very attractive feature, from my point of view, that it offers the possibility of allowing unique predictions even if the underlying string theory does not have a unique vacuum. I have also emphasized, however, that there are important problems in understanding the implications of eternal inflation. First, there is the problem that we do not know how to treat the situation in which the scalar field climbs upward to the Planck energy scale. Second, the definition of probabilities in an eternally inflating spacetime is not yet a closed issue, although important progress has been made. And third, I might add that the entire present approach is at best semiclassical. A better treatment may not be possible until we have a much better handle on quantum gravity, but eventually this issue will have to be faced.

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