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Neutrinos and supernova theory

Adam Burrows*, Timothy Young

Department of Astronomy, The University of Arizona, Tucson, AZ 85721, USA

Abstract

Neutrinos are the primary agents in core-collapse supernova explosions and their signature in underground terrestrial detectors should bear the stamp of the events that launched the explosion and gave birth to either a neutron star or a black hole. In this paper, we outline the neutrino burst, discuss some suggestive systematics with progenitor mass, review the evidence for asymmetries in supernova explosions, and speculate about pulsar kicks. Moreover, we summarize new calculations concerning inelastic neutrino–nucleon scattering and nucleon–nucleon bremsstrahlung. The latter processes are important to the emergent ν_μ neutrino spectra and incorporate some sweet physics that Dave Schramm would no doubt have loved to explore. © 2000 Elsevier Science B.V. All rights reserved.

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1. Basics of the neutrino signature of stellar collapse

Neutrinos are the major signatures of the inner turmoil of the dense core of the massive star and they carry away the binding energy of the young neutron star, a full 10% of its mass energy. The detection of collapse neutrinos, their “light curve” and spectra, will allow us to follow in real time the phenomena of stellar death and birth. The supernova, SN1987A, provided a glimpse of what might be possible, but it yielded only 19 events; we can expect the current generation of underground neutrino telescopes to collect thousands of events from a galactic supernova.

There is a broad consensus on the basic features of the neutrino light curve from a supernova [1], but it should be recalled that the luminosities and timescales for different massive star progenitors

* Corresponding author.

E-mail address: burrows@as.arizona.edu (A. Burrows).

will be different. Generically, infall may last from 200 to 600 ms during which time electron neutrinos will predominate. They will have roughly a capture spectrum that gradually hardens until shock breakout. The rise time of the associated luminosity depends upon the nuclear symmetry energy, but is approximately 5 ms. The total energy radiated during this phase is roughly 10^{51} erg. Bounce is almost immediately followed by the formation of the shock in the neutrino-opaque regions (at near 20 km). The shock starts with a velocity near 50 000 km/s and so very quickly achieves the neutrinosphere (50–100 km) and breaks out. Shock breakout is announced by a prodigious burst of electron neutrinos produced by electron capture on free protons newly liberated by shock dissociation of the infalling nuclei. The electron neutrino luminosity may achieve 10^{54} erg s⁻¹. The characteristic time of the breakout burst is 3–10 ms and the total energy radiated in electron-type neutrinos during breakout is $\sim 3 \times 10^{51}$ erg. The magnitude of the latter will depend on the density structure of the collapsing Chandrasekhar core and will be higher for the more massive progenitors. During this phase, perhaps 10 events in both SuperK and in ICARUS can be expected from a collapse at 10 kpc.

During breakout, the matter is heated to such a degree that $\bar{\nu}_e$ neutrinos and ν_μ and ν_τ neutrinos and anti-neutrinos (hereafter “ $\nu_\mu s$ ”) are thermally produced and radiated. The turn-on timescale of this component is less than 1 ms, but the initial luminosity of the $\bar{\nu}_e s$ and the $\nu_\mu s$ depends upon the degree of degeneracy of the electrons near the neutrinospheres and the magnitude of the production sources, still poorly known. It is thought that the initial $\bar{\nu}_e$ neutrino luminosity is within about one order of magnitude of its peak value (~ 20 – 50 ms after breakout). Even at such a level, at 10 kpc both SuperK and SNO will register 100’s of $\bar{\nu}_e$ events per second, in SuperK perhaps a kilohertz. After the abrupt rise, the $\bar{\nu}_e$ neutrino luminosity rises further to approximately meet the falling ν_e luminosity. After 20–50 ms, the two decay together as the light curve transitions to the longer-term proton-neutron star cooling and neutronization phase. Similarly, the ν_μ neutrino luminosity per species achieves a value not more than 30% away from the electron neutrino luminosity.

The decay is gradual and there may be some quasi-periodic pulsation of the luminosities during this phase. However, the shock wave launched with such fanfare stalls into an accretion shock at 100–200 km within 10–20 ms of breakout. There is a delay to explosion, that may last between a few hundred milliseconds and a second, during which time perhaps $\geq 10^{53}$ erg of neutrinos may be radiated. When it comes, explosion should be accompanied by a decrease by about a factor of two over about 20 ms in all the neutrino luminosities. This may be detectable. After explosion, the luminosities decay on timescales of seconds to a minute. Indeed, after as long as a minute, the event rate at 10 kpc in SuperK may still be as high as one per second. After breakout, the spectra of all the neutrino species first harden on timescales of hundreds of milliseconds, then soften, particularly after explosion, as the luminosity inexorably decays. The rise and fall timescales, as well as the explosion time, are not known theoretically with sufficient precision.

Hence, the important features detectors should key in on are: the infall rise, the breakout, the early $\bar{\nu}_e$ neutrino rise, the production of $\nu_\mu s$, the signature of explosion, the rise and fall of the average neutrino energies, and the late-time persistence. In addition, if a black hole forms during the high-luminosity phase, the prediction is that the signal will stop within less than a millisecond. Such a phenomenon will be detectable.

Given this generic neutrino light curve, can we use accurate timing of the features in the burst to triangulate on the supernova? This will depend upon the signal strength (and, hence, the distance).

At the canonical distance of 10 kpc, with the forward-peaked SuperK ν_e neutrino events (~ 100 – 150), one should be able to achieve $\sim 4^\circ$ pointing without the aid of the network. If the initial $\bar{\nu}_e$ signal is indeed as abrupt as we believe and if it starts at a high luminosity, then initial count rates near 1 kHz in SuperK, SNO, and LVD/MACRO/ICARUS might enable the network to locate a supernova to within $\sim 10^\circ$ at 10 kpc. The fact that the current detectors are all in the northern hemisphere is a problem, as is the possible fuzziness of the initial luminosity rise. Furthermore, there is general excitement that the network of neutrino telescopes now being established underground might indeed be able to announce, with whatever angular precision, the advent of a galactic supernova and allow the astronomical community the early warning it has never before enjoyed.

2. Core-collapse supernova theory and suggestive systematics

All groups that do multi-D hydrodynamic modeling of supernovae obtain vigorous convection in the semi-transparent mantle bounded by the stalled shock [2–7]. There is a consensus that the neutrinos drive the explosion [8] after a delay whose magnitude has yet to be determined, but that may be between 100 and 1000 ms. Whether any convective motion or hydrodynamic instability is central to the explosion mechanism is not clear, with five groups [2–4,6,7] voting yes or maybe and one group [5] voting no.

Hence, and unfortunately, theory is not yet adequate to determine the systematics with progenitor mass of the explosion energies, residue masses, ^{56}Ni yields, kicks, or, in fact, almost any parameter of a real supernova explosion. Despite this, there are hints, both observational and theoretical. The gravitational binding energy (BE) exterior to a given interior mass is an increasing function of progenitor mass, ranging at $1.5M_\odot$ interior mass from about 10^{50} erg for a $10M_\odot$ progenitor to as much as 3×10^{51} erg for a $40M_\odot$ progenitor [2,9]. This large range must affect the viability of explosion and its energy. It is not unreasonable to conclude, in a very crude way, that BE sets the scale for the supernova explosion energy. When the “available” energy exceeds the “necessary” binding energy, both very poorly defined quantities, explosion is more “likely”. However, how does the supernova, launched in the inner protoneutron star, know what binding energy it will be called upon to overcome when achieving larger radii? Since the post-bounce, pre-explosion accretion rate (\dot{M}) is a function of the star’s inner density profile, as is the inner BE, and since a large \dot{M} seems to inhibit explosion, it may be via \dot{M} that BE, at least that of the inner star, is sensed. Furthermore, a neutrino-driven explosion requires a neutrino-absorbing mass and there is more mass available in the denser core of a more massive progenitor. One might think that binding energy and absorbing mass partially compensate or that a more massive progenitor just can wait longer to explode, until its binding energy problems are buried in the protoneutron star and \dot{M} has subsided. The net effect in both cases may be similar explosion energies for different progenitors, though the residue mass could be systematically higher for the more massive stars. However, if these effects do not compensate, the fact that binding energy and absorbing mass are increasing functions of progenitor mass hints that the supernova explosion energy may also be an increasing function of mass. Since BE varies so much along the progenitor continuum, the range in the explosion energy may not be small. Curiously, the amount of ^{56}Ni produced explosively also depends upon the mass between the residue and the radius at which the shock temperature goes

below the explosive Si-burning temperature, a radius that depends upon explosion energy. Hence, the amount of ^{56}Ni produced may also increase with progenitor mass. Thermonuclear energy only partially compensates for the binding energy to be overcome, the former being about 10^{50} erg for every $0.1M_{\odot}$ of ^{56}Ni produced.

Not all ^{56}Ni produced need be ejected. Fallback is possible and whether there is significant fallback must depend upon the binding energy profile. We think that there is not much fallback for the lighter progenitors, perhaps for masses below $15M_{\odot}$, but that there is significant fallback for the heaviest progenitors. The transition between the two classes may be abrupt. We base this surmise on the miniscule binding energies and tenuous envelopes of the lightest massive stars and on the theoretical prejudice that the r-process, or some fraction of it, originates in the proton-neutron winds that follow the explosion for the lightest massive stars [10,11]. If there were significant fallback, these winds and their products would be smothered.

If there is significant fallback, the supernova may be in jeopardy and much of the ^{56}Ni produced will reimplode. There may be a narrow range of progenitor mass over which the supernova is still viable, while fallback is significant and both the mass of ^{56}Ni ejected and the supernova energy are decreasing. Above this mass range, a black hole may form. Hence, both low- and high-mass supernova progenitors may have low ^{56}Ni yields. Recently, two Type IIp supernovae have been detected, SN1994W [12] and SN1997D [13], which have very low ^{56}Ni yields ($\leq 0.0026M_{\odot}$ and $\leq 0.002M_{\odot}$, respectively), long-duration plateaus, and large inferred ejecta masses ($\geq 25M_{\odot}$). The estimated explosion energy for SN1997D is a slight 0.4×10^{51} erg. (SN1987A's explosion energy was $1.5 \pm 0.5 \times 10^{51}$ erg and its ^{56}Ni yield was $0.07M_{\odot}$.) These two supernovae may reside in the fallback gap and imply that the black hole cut-off is near $30M_{\odot}$. However, recently Chugai and Utrobin [14] have reinterpreted the light curve and oxygen yield of SN1997D to imply that its progenitor was a lower-mass massive star (perhaps $8\text{--}10M_{\odot}$). In addition, Brown et al. [15] point out that because convective carbon burning is skipped around a ZAMS mass of $20M_{\odot}$ (as a consequence of which there is a jump in the iron core mass), $20M_{\odot}$ may be the natural bifurcation point between black holes and neutron stars. Be that as it may, there is now evidence that the nickel yields and explosion energies of supernovae span a wide range and that both may be small near the mass boundaries of the progenitor regime.

In sum, supernova ^{56}Ni yields may vary by a factor of ~ 100 and may peak at some intermediate progenitor mass, the supernova explosion energy may vary by a factor of ~ 10 and also may peak at some intermediate progenitor mass, and the black hole cut-off mass may be near $30M_{\odot}$. However, and importantly, whether real theoretical calculations will bear out these hinted-at systematics is as yet very unclear.

3. Asymmetries of supernova explosions

There are many observational indications that supernova explosions are indeed aspherical. Fabry–Perot spectroscopy of the young supernova remnant Cas A, formed around 1680 AD, reveals that its calcium, sulfur, and oxygen element distributions are clumped and have gross back–front asymmetries [16]. No simple shells are seen. Many supernova remnants, such as N132D, Cas A, E0102.2-7219, and SN0540-69.3, have systemic velocities relative to the local ISM of up to 900 km s^{-1} [17]. X-ray data taken by ROSAT of the Vela remnant reveal bits of shrapnel

with bow shocks [18]. The supernova, SN1987A, is a case study in asphericity: (1) its X-ray, gamma-ray, and optical fluxes and light curves require that shards of the radioactive isotope ^{56}Ni were flung far from the core in which they were created, (2) the infrared line profiles of its oxygen, iron, cobalt, nickel, and hydrogen are ragged and show a pronounced red–blue asymmetry, (3) its light is polarized and (4) recent Hubble Space Telescope pictures of its inner debris reveal large clumps and hint at a preferred direction [19]. Furthermore, radio pictures of the supernova SN1993J, which also has polarized optical spectral features, depict a broken shell. One of the most intriguing recent finds is the supernova SN1997X, which is a so-called Type Ic explosion. This supernova shows the greatest optical polarization of any to date (Lifan Wang, private communication). Type Ic supernovae are thought to be explosions of the bare carbon/oxygen cores of massive star progenitors stripped of their envelopes and some may be connected to a fraction of γ -ray bursts, for which jets have been inferred. As such, SN1997X's large polarization implies that the inner supernova cores, and, hence, the explosions themselves, are fundamentally asymmetrical. No doubt, instabilities in the outer envelopes of supernova progenitors clump and mix debris clouds and shatter spherical shells. The observation of hydrogen deep in SN1987A's ejecta [20] strongly suggests the work of such mantle instabilities. However, the data collectively, particularly for the heavier elements produced in the inner core, are pointing to asymmetries in the central engine of explosion itself.

4. Neutron star kicks

Strong evidence that neutron stars experience a net kick at birth has been mounting for years. In 1993 [21,22], it was demonstrated that the pulsars are the fastest population in the galaxy ($\langle v \rangle \sim 450 \text{ km s}^{-1}$). Such speeds are far larger than can result generically from orbital motion due to birth in a binary (the “so-called” Blaauw effect). An extra “kick” is required, probably during the supernova explosion itself [23]. In the pulsar binaries, PSR J0045-7319 and PSR 1913 + 16, the spin axes and the orbital axes are misaligned, suggesting that the explosions that created the pulsars were not spherical [24,25]. In fact, for the former the orbital motion seems retrograde relative to the spin [26] and the explosion may have kicked the pulsar backwards. In addition, the orbital eccentricities of Be star/pulsar binaries are higher than one would expect from a spherical explosion, also implying an extra kick [27]. Furthermore, low-mass X-ray binaries (LMXB) are bound neutron star/low-mass star systems that would have been completely disrupted during the supernova explosion that left the neutron star, had that explosion been spherical [28]. In those few cases, a countervailing kick may have been required to keep the system bound. The kick had to act on a timescale shorter than the orbit period and the explosion orbit crossing time. Otherwise, the process would have been uselessly adiabatic. One is tempted to evoke as further proof the fact that pulsars seen around young (age $\leq 10^4$ years) supernova remnants are on average far from the remnant centers, but here ambiguities in the pulsar ages and distances and legitimate questions concerning the reality of many of the associations make this argument rather less convincing [29,30]. However, the ROSAT observations of the 3700 year-old supernova remnant Puppis A show an X-ray spot that has been interpreted as its neutron star [31]. This object has a large X-ray to optical flux ratio, but no pulsations are seen. If this interpretation is legitimate, then the inferred neutron star transverse speed is $\sim 1000 \text{ km s}^{-1}$. Interestingly, the spot is opposite to

the position of the fast, oxygen-rich knots, as one might expect in some models of neutron star recoil during the supernova explosion. Whatever the correct interpretation of the Puppis A data, it is clear that many neutron stars are given a hefty extra kick at birth (though the distribution of these kicks is broad) and that it is reasonable to implicate asymmetries in the supernova explosion itself.

4.1. *A theoretical aside on pulsar kicks*

Supernova theorists have determined that protoneutron star/supernova cores are indeed grossly unstable to Rayleigh–Taylor-like instabilities [2–4]. During the post-bounce delay to explosion that might last 100–1000 ms, these cores with 100–200 km radii are strongly convective, boiling and churning at sonic ($\sim 3 \times 10^4 \text{ km s}^{-1}$) speeds. Any slight asymmetry in collapse can amplify this jostling and result in vigorous kicks and torques [2,32,33] to the residue that can be either systematic or stochastic. Whatever the details, it would seem odd if the nascent neutron star were not left with a net recoil and spin, though whether pulsar speeds as high as 1500 km s^{-1} (cf. the Guitar Nebula) can be reached through this mechanism is unknown. Furthermore, asymmetries in the matter field may result in asymmetries in the emission of the neutrinos that carry away most of the binding energy of the neutron star. A net angular asymmetry in the neutrino radiation of only 1% would give the residue a recoil of $\sim 300 \text{ km s}^{-1}$. Not surprisingly, many theorists have focussed on producing such a net asymmetry in the neutrino field, either evoking anisotropic accretion, exotic neutrino flavor physics, or the influence of strong magnetic fields on neutrino cross sections and transport. The latter is particularly interesting, but generally requires magnetic fields of 10^{14} – 10^{16} G [34], far larger than the canonical pulsar surface field of 10^{12} G . Perhaps, the pre-explosion convective motions themselves can generate via dynamo action the required fields. Perhaps, these fields are transient and subside to the observed fields after the agitation of the explosive phase. It would be hard to hide large fields of 10^{15} G in the inner core of an old neutron star, while still maintaining standard surface fields of 10^{12} G . In this context, it is interesting to note that surface fields as high as 10^{15} G are very indirectly being inferred for the so-called soft-gamma repeaters [35], but these are a small fraction of all neutron stars. If such large fields are necessary to impart, via anisotropic neutrino emission, the kicks observed, then the coincidence that Spruit and Phinney [33] note between the fields needed to enforce slow pre-collapse rotation and those observed in pulsars after flux freezing amplification is of less significance.

Whether the kick mechanism is hydrodynamic or due to neutrino momentum, one might expect that the more massive progenitors would give birth to speedier neutron stars. More massive progenitors generally have more massive cores. If the kick mechanism relies on the anisotropic ejection of matter [32], then for a given explosion energy and degree of anisotropy we might expect the core ejecta mass and, hence, the dipole component of the ejecta momentum to be larger (“ $p \sim \sqrt{2ME}$ ”), resulting in a larger kick. The explosion energy itself may also be larger for the more massive progenitors, enhancing the effect. If the mechanism relies on anisotropic neutrino emission, the residues of more massive progenitors are likely to be more massive and have a greater binding energy ($E_B \propto M_{\text{NS}}^2$) to radiate. Hence, for a given degree of neutrino anisotropy, the impulse and kick ($\propto E_B/M_{\text{NS}}$) would be greater. In either case, despite the primitive nature of our current

understanding of kick mechanisms, given the above arguments it is not unreasonable to speculate that the heaviest massive stars might yield the fastest neutron stars.

5. New ideas in neutrino–matter interactions

Over the years, neutrino transport theory and the associated microphysics have reached a sophisticated level of refinement [36–43]. However, despite these efforts, recent progress in modeling supernovae, and new insights gained into the character of multi-dimensional neutrino-driven explosions [2–5], the supernova explosion problem is not solved in detail.

Neutrino–matter cross sections, both for scattering and for absorption, play the central role in neutrino transport. The major processes are the super-allowed charged-current absorptions of ν_e and $\bar{\nu}_e$ neutrinos on free nucleons, neutral-current scattering off of free nucleons, alpha particles, and nuclei [44], neutrino–electron/positron scattering, neutrino–nucleus absorption, neutrino–neutrino scattering, neutrino–antineutrino absorption, and the inverses of various neutrino production processes such as nucleon–nucleon bremsstrahlung and the modified URCA process ($\nu_e + n + n \rightarrow e^- + p + n$). Compared with photon–matter interactions, neutrino–matter interactions are relatively simple functions of incident neutrino energy. Resonances play little or no role and continuum processes dominate. Nice summaries of the various neutrino cross sections of relevance in supernova theory are given in Tubbs and Schramm [36] and in Bruenn [40]. Below, we summarize two cutting-edge topics that have of late assumed new importance in the study of neutrino–matter interactions. In Section 5.1, we provide some straightforward formulae that can be used to properly handle inelastic scattering off of nucleons in the atmospheres of protoneutron stars. We also discuss the possible effect of many-body correlations on the magnitude of the neutrino–nucleon scattering rates at high densities. If the delay to explosion is more than about one second, the inferred suppression of these cross sections may have consequences for the neutrino-driven mechanism itself. In Section 5.2 we discuss nucleon–nucleon bremsstrahlung, a process that can compete with pair annihilation as a source for ν_μ , $\bar{\nu}_\mu$, ν_τ and $\bar{\nu}_\tau$ neutrinos.

5.1. Dynamic structure factors for neutrino–nucleon interactions

Previously, it had been assumed that neutrino–nucleon scattering was elastic [45]. However, recent reappraisals reveal that the product of the underestimated energy transfer per neutrino–nucleon scattering with cross section exceeds the corresponding quantity for neutrino–electron scattering. Since ν_e and $\bar{\nu}_e$ neutrinos participate in super-allowed charged-current absorptions on nucleons, neutrino–nucleon scattering has little effect on their rate of equilibration. However, such scattering would seem to be important for ν_μ and ν_τ equilibration. Many-body correlation suppressions appear only above neutrinosphere densities ($\sim 10^{11}$ – 10^{13} g cm^{−3}). Hence, it is only the kinematic effect, and not the interaction effect, that need be considered when studying the emergent spectra. Without interactions, $S(q, \omega)$, the dynamical structure factor for neutrino–nucleon scattering, is simply

$$S(q, \omega) = 2 \int \frac{d^3p}{(2\pi)^3} \mathcal{F}(|\mathbf{p}|) (1 - \mathcal{F}(|\mathbf{p} + \mathbf{q}|)) 2\pi \delta(\omega + \varepsilon_p - \varepsilon_{p+q}), \quad (1)$$

where $\mathcal{F}(|\mathbf{p}|)$ is the nucleon Fermi–Dirac distribution function, ε_p is the nucleon energy, ω is the energy transfer to the medium, and \mathbf{q} is the momentum transfer. The magnitude of \mathbf{q} is related to ω and E_1 , the incident neutrino energy, through the neutrino scattering angle, θ , by the expression

$$q = [E_1^2 + (E_1 - \omega)^2 - 2E_1(E_1 - \omega)\cos\theta]^{1/2}. \quad (2)$$

In the elastic limit and ignoring final-state nucleon blocking, $S(q, \omega) = 2\pi\delta(\omega)n_n$, the expected result, where n_n is the nucleon's number density.

The neutral current scattering rate of either neutrons or protons is [46]

$$\frac{d^2\Gamma}{d\omega d\cos\theta} = (4\pi^2)^{-1}G_W^2(E_1 - \omega)^2[1 - \mathcal{F}_v(E_1 - \omega)]\mathcal{I}_{\text{NC}}, \quad (3)$$

where

$$\mathcal{I}_{\text{NC}} = [(1 + \cos\theta)V + (3 - \cos\theta)A]S(q, \omega) \quad (4)$$

and

$$S(q, \omega) = 2\text{Im}\Pi^{(0)}(1 - e^{-\beta\omega})^{-1}. \quad (5)$$

V and A are the applicable vector and axial-vector coupling terms and $\beta = 1/kT$. The free polarization function, $\Pi^{(0)}$, contains the full kinematics of the scattering, as well as blocking due to the final-state nucleon, and the relevant imaginary part of $\Pi^{(0)}$ is given by

$$\text{Im}\Pi^{(0)}(q, \omega) = (m^2/2\pi q\beta)\log[1 + e^{-Q_\pm^2 + \beta\mu}/1 + e^{-Q_\pm^2 + \beta\mu - \beta\omega}], \quad (6)$$

where

$$Q_\pm = (m\beta/2)^{1/2}(\mp \omega/q + q/2m), \quad (7)$$

μ is the nucleon chemical potential, and m is the nucleon mass. The dynamical structure factor, $S(q, \omega)$, contains all of the information necessary to handle angular and energy redistribution due to scattering.

In the non-degenerate nucleon limit, Eq. (6) can be expanded to lowest order in Q_\pm^2 to obtain, using Eq. (5), an approximation to the dynamical structure factor:

$$S(q, \omega) = (n(2\pi m\beta)^{1/2}/q)e^{-Q_\pm^2}, \quad (8)$$

where n is the nucleon number density. This says that for a given momentum transfer the dynamical structure factor is approximately a Gaussian in ω .

For the charged-current absorption process, $\nu_e + n \rightarrow e^- + p$, $\text{Im}\Pi^{(0)}(q, \omega)$ is given by a similar expression:

$$\text{Im}\Pi^{(0)}(q, \omega) = (m^2/2\pi\beta q)\log[1 + e^{-Q_\pm^2 + \beta\mu_n}/1 + e^{-Q_\pm^2 + \beta\mu_p - \beta\omega}]. \quad (9)$$

Eq. (9) inserted into Eq. (5) with a $(1 - e^{-\beta(\omega + \hat{\mu})})$, as is appropriate for the charged-current process, substituted for $(1 - e^{-\beta\omega})$, results in an expression that is a bit more general than the one employed to date by most practitioners, i.e., $S = (X_n - X_p)/(1 - e^{-\hat{\mu}/T})$. In the non-degenerate nucleon limit, the structure factor for the charged-current process can be approximated by Eq. (8) with $n = n_n$. Note that for the structure factor of a charged-current interaction one must distinguish between the

initial- and the final-state nucleons and, hence, between their chemical potentials. To obtain the structure factor for the $\bar{\nu}_e$ absorption process, one simply permutes μ_n and μ_p in Eq. (9) and substitutes $-\hat{\mu}$ for $\hat{\mu}$ in the $(1 - e^{-\beta(\omega + \hat{\mu})})$ term.

However, including correlations due to nucleon–nucleon interactions, indications are that we have been overestimating the neutral-current and the charged-current cross sections above $10^{14} \text{ g cm}^{-3}$ by factors of from 2 to 5, depending upon density and the equation of state [46–49]. The many-body interaction corrections increase with density, decrease with temperature, and for neutral-current scattering are roughly independent of incident neutrino energy. Furthermore, the spectrum of energy transfers in neutrino scattering is considerably broadened by the interactions in the medium. An identifiable component of this broadening comes from the absorption and emission of quanta of collective modes akin to the Gamow–Teller and Giant-Dipole resonances in nuclei (zero-sound; spin sound), with Čerenkov kinematics. This implies that all scattering processes may need to be handled with the full energy redistribution formalism and that ν -matter scattering at high densities can not be considered elastic. One consequence of this reevaluation is that the late-time ($\geq 500 \text{ ms}$) neutrino luminosities may be as much as 50% larger for more than a second than heretofore estimated. These luminosities reflect more the deep protoneutron star interiors than the early post-bounce luminosities of the outer mantle and the accretion phase. Since neutrinos drive the explosion, this may have a bearing on the specifics of the supernova mechanism.

5.2. Nucleon–nucleon bremsstrahlung

A production process for neutrino/anti-neutrino pairs that has received but little attention to date in the supernova context is neutral-current nucleon–nucleon bremsstrahlung ($n_1 + n_2 \rightarrow n_3 + n_4 + \nu\bar{\nu}$). Its importance in the cooling of old neutron stars, for which the nucleons are quite degenerate, has been recognized for years [50], but only in the last few years has it been studied for its potential importance in the atmospheres of protoneutron stars and supernovae [51–53]. Neutron–neutron, proton–proton, and neutron–proton bremsstrahlung are all important, with the latter the most important for symmetric matter. As a source of ν_e and $\bar{\nu}_e$ neutrinos, nucleon–nucleon bremsstrahlung can not compete with the charged-current capture processes. However, for a range of temperatures and densities realized in supernova cores, it may compete with e^+e^- annihilation as a source for $\nu_\mu, \bar{\nu}_\mu, \nu_\tau$ and $\bar{\nu}_\tau$ neutrinos (“ ν_μ ”s). The major obstacles to obtaining accurate estimates of the emissivity of this process are an understandable reticence to include the full and proper nucleon–nucleon potentials, uncertainty concerning the degree of suitability of the Born Approximation, and ignorance concerning the true role of many-body effects [51,54,55]. Since the nucleons in protoneutron star atmospheres are not degenerate, we present some results from Burrows et al. [56] for the total and differential emissivities of this process in that limit, assuming a one-pion exchange (OPE) potential model to calculate the nuclear matrix element and using a fudge factor (ζ) to subsume all ignorance.

Our focus is on obtaining a useful single-neutrino final-state emission (source) spectrum, as well as a final-state pair energy spectrum and the total emission rate. The necessary ingredients are the matrix element for the interaction and a workable procedure for handling the phase space terms, constrained by the conservation laws. Burrows et al. [56] follow Brinkmann and Turner [55] for both of these elements. In particular, they assume for the $n + n \rightarrow n + n + \nu\bar{\nu}$ process that the

matrix element is

$$\sum_s |\mathcal{M}|^2 = \frac{64}{4} G^2 (f/m_\pi)^4 g_A^2 \left[\left(\frac{k^2}{k^2 + m_\pi^2} \right)^2 + \dots \right] \frac{\omega_\nu \omega_{\bar{\nu}}}{\omega^2} = A \frac{\omega_\nu \omega_{\bar{\nu}}}{\omega^2}, \quad (10)$$

where the 4 in the denominator accounts for the spin average for identical nucleons, G is the weak coupling constant, f (~ 1.0) is the pion–nucleon coupling constant, g_A is the axial-vector coupling constant, the term in brackets is from the OPE propagator plus exchange and cross terms, k is the nucleon momentum transfer, and m_π is the pion mass. In Eq. (10), they have dropped $\mathbf{q}_\nu \cdot \mathbf{k}$ terms from the weak part of the total matrix element. To further simplify the calculation, they set the “propagator” equal to a constant ζ , a number of order unity, and absorb into ζ all interaction ambiguities. The constant A in Eq. (10) remains.

Inserting a $\int \delta(\omega - \omega_\nu - \omega_{\bar{\nu}}) d\omega$ by the neutrino phase space terms times $\omega \omega_\nu \omega_{\bar{\nu}} / \omega^2$ and integrating over $\omega_{\bar{\nu}}$ yields

$$\int \omega \frac{\omega_\nu \omega_{\bar{\nu}}}{\omega^2} \frac{d^3 \mathbf{q}_\nu}{(2\pi)^3 2\omega_\nu} \frac{d^3 \mathbf{q}_{\bar{\nu}}}{(2\pi)^3 2\omega_{\bar{\nu}}} \rightarrow \frac{1}{(2\pi)^4} \int_0^\infty \int_0^\omega \frac{\omega_{\bar{\nu}}^2 (\omega - \omega_{\bar{\nu}})^2}{\omega} d\omega_\nu d\omega, \quad (11)$$

where ω equals $(\omega_\nu + \omega_{\bar{\nu}})$. If we integrate over ω_ν , we can derive the ω spectrum. A further integration over ω will result in the total volumetric energy emission rate. If we delay such an integration, after the nucleon phase space sector has been reduced to a function of ω and if we multiply Eq. (11) by ω_ν / ω , an integration over ω from ω_ν to infinity will leave the emission spectrum for the single final-state neutrino. This is of central use in multi-energy group transport calculations and with this differential emissivity and Kirchoff’s Law we can derive an absorptive opacity.

Whatever our final goal, we need to reduce the nucleon phase space integrals and to do this, Burrows et al. [56] use the coordinates and approach of Brinkmann and Turner [55]. They define new momenta: $p_+ = (p_1 + p_2)/2$, $p_- = (p_1 - p_2)/2$, $p_{3c} = p_3 - p_+$, and $p_{4c} = p_4 - p_+$, where nucleons 1 and 2 are in the initial state. Useful direction cosines are $\gamma_1 = p_+ p_- / |p_+| |p_-|$ and $\gamma_c = p_+ p_{3c} / |p_+| |p_{3c}|$. Defining $u_i = p_i^2 / 2mT$ and using energy and momentum conservation, one can show that

$$\begin{aligned} d^3 p_1 d^3 p_2 &= 8 d^3 p_+ d^3 p_- , \\ \omega &= 2T(u_- - u_{3c}) , \\ u_{1,2} &= u_+ + u_- \pm 2(u_+ u_-)^{1/2} \gamma_1 , \\ u_{3,4} &= u_+ + u_{3c} \pm 2(u_+ u_{3c})^{1/2} \gamma_c . \end{aligned} \quad (12)$$

In the non-degenerate limit, the $\mathcal{F}_1 \mathcal{F}_2 (1 - \mathcal{F}_3)(1 - \mathcal{F}_4)$ term reduces to $e^{2y} e^{-(u_+ + u_-)}$, where y is the nucleon degeneracy factor. Using Eq. (12), we see that the quantity $(u_+ + u_-)$ is independent of both γ_1 and γ_c . This is a great simplification and makes the angle integrations trivial. Annihilating $d^3 p_4$ with the momentum delta function in Fermi’s Golden Rule, noting that $p_i^2 dp = [(2mT)^{1/2}/2] u_i^{1/2} du_i$, pairing the remaining energy delta function with u_- , and integrating u_+

from 0 to ∞ , we obtain

$$dQ_{nb} = \frac{Am^{4.5}}{2^8 \times 3 \times 5\pi^{8.5}} T^{7.5} e^{2y} e^{-\omega/T} (\omega/T)^4 \left[\int_0^\infty e^{-x(x^2 + x\omega/T)^{1/2}} dx \right] d\omega. \quad (13)$$

The variable x over which we are integrating in Eq. (13) is equal to $2u_{3c}$. The integral is analytic and yields

$$\int_0^\infty e^{-x(x^2 + x\omega/T)^{1/2}} dx = \eta e^\eta K_1(\eta), \quad (14)$$

where K_1 is the standard modified Bessel function of imaginary argument, related to the Hankel functions, and $\eta = \omega/2T$. Hence, the ω spectrum is given by

$$\frac{dQ_{nb}}{d\omega} \propto e^{-\omega/2T} \omega^5 K_1(\omega/2T). \quad (15)$$

It can easily be shown that $\langle \omega \rangle = 4.364T$ [54]. Integrating Eq. (13) over ω and using the thermodynamic identity in the non-degenerate limit:

$$e^y = \left(\frac{2\pi}{mT} \right)^{3/2} n_n/2, \quad (16)$$

where n_n is the density of neutrons (in this case), one derives for the total neutron–neutron bremsstrahlung emissivity of a single neutrino pair

$$Q_{nb} = 2.08 \times 10^{30} \zeta (X_n \rho_{14})^2 (T/\text{MeV})^{5.5} \text{erg cm}^{-3} \text{s}^{-1}, \quad (17)$$

where ρ_{14} is the mass density in units of $10^{14} \text{ gm cm}^{-3}$ and X_n is the neutron mass fraction. Interestingly, this is within 30% of the result in Suzuki [53], even though he has substituted, without much justification, $(1 + \omega/2T)$ for the integral in Eq. (13). The proton–proton and neutron–proton processes can be handled similarly and the total bremsstrahlung rate is then obtained by substituting $X_n^2 + X_p^2 + \frac{14}{3} X_n X_p$ for X_n^2 in Eq. (17) [55]. At $X_n = X_p = 0.5$, taking the ratio of augmented Eq. (17) to the pair annihilation rate, one obtains the promising ratio: $\sim 2\zeta \rho_{13}^2 (6 \text{ MeV}/T)^{3.5}$. Setting the correction factor ζ equal to ~ 0.5 [51], we find near and just below the ν_μ neutrinosphere, that bremsstrahlung may be comparable to classical pair production.

If in Eq. (11) one does not integrate over ω_v , but at the end of the calculation one integrates over ω from ω_v to ∞ , after some manipulation one obtains the single neutrino emissivity spectrum

$$\frac{dQ_{nb}}{d\omega_v} = 2C(Q_{nb}/T^4) \omega_v^3 \int_1^\infty \frac{e^{-2\eta_v \xi}}{\xi^3} (\xi^2 - \xi)^{1/2} d\xi, \quad (18)$$

where $\eta_v = \omega_v/2T$, C is the normalization constant equal to $(3 \times 5 \times 7 \times 11)/2^{11}$ ($\cong 0.564$), and we have used the integral representation of $K_1(\eta)$ and reversed the order of integration. In Eq. (18), Q_{nb} is the emissivity for the pair.

Eq. (18) is the approximate neutrino emission spectrum due to nucleon–nucleon bremsstrahlung. A useful fit to Eq. (18), good to better than 3% over the full range of important values of η_ν , is

$$\frac{dQ_{nb}}{d\omega_\nu} \simeq \frac{0.234Q_{nb}}{T} \left(\frac{\omega_\nu}{T} \right)^{2.4} e^{-1.1\omega_\nu/T}. \quad (19)$$

6. Conclusions

Even after 40 years of progress and development, we are far from a systematic and detailed understanding of the core-collapse supernova mechanism. To be sure, the subject has gotten much richer, the numerical tools have gotten much better, and many insights have been won. In addition, there are hints at connections between some supernovae and some gamma-ray bursts, providing yet another astrophysical context in which the neutrino and its interactions may be crucial. However, as we approach the new millenium the mind beggars at the number of basic questions with which we are still groping. Dave Schramm was a pioneer in the modern study of supernova neutrinos and as we renew our focus in the next century on this perennial frontier of high-energy astrophysics, we rededicate ourselves to this, one of Dave's very favorite puzzles.

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