

Astronomy 45
Introduction to Astrophysics

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Alex Dalgarno

(adalgarno@cfa.harvard.edu)

Astronomy 45 is an introduction to the concepts and methods of astrophysics. It includes a survey of astronomical objects, a description of astronomical measurements and units and an introductory account of the astrophysics of radiation, orbital motions, tidal interactions, stellar structure, energy generation, many-body dynamics and cosmology.

1.1 Introduction

By astronomy we usually mean the observation and measurement of the properties of extraterrestrial objects. The observations are almost entirely of electromagnetic radiation over an extended wavelength range through the radio, millimeter, infrared, optical and ultraviolet to X-rays and γ -rays. Information is also obtained from studies of cosmic rays and neutrinos. The measurement data include positions, velocities, brightnesses and spectra of objects which include the planets, the Sun, stars, globular clusters, nebulae, novae, supernovae, the interstellar medium, galaxies, clusters of galaxies, neutron stars and black holes. Astrophysics applies

physics to interpret and understand the observations and it attempts to develop a unified picture of the history and evolution of the Universe.

Astrophysics is characterized by enormous changes of scale from the size of nuclei 10^{-13} cm to the distances to the earliest galaxies 10^{41} cm. The different scales can be hard to grasp and we have to find ways to manage them conceptually. One advantage Astrophysics has over Physics and Chemistry is that high precision is often unnecessary. Estimates correct to an order of magnitude often suffice.

Astrophysics covers a wide range of physics and we need some knowledge of many-body dynamics, of atomic, molecular and optical physics, nuclear physics, plasma physics, condensed matter physics, particle physics and also chemistry. The participation of these areas of physics makes the study of astrophysics interesting and challenging.

I will begin by describing some pre-astrophysics history, starting with the Solar system and the planets.

1.2 Planets

Viewed from Earth, the Sun appears to move eastward with respect to the stars and circles the sky in one year (Fig. 1-1). The imaginary path it traces out is called the *ecliptic*. The N-S polar axis of the Earth points in a fixed direction in space or nearly so. Perpendicular to it through the center of the Earth is the equatorial plane. Imagine the plane to be extending in space. The equatorial plane of the Earth is inclined by 23.5° degrees with respect to the plane of the ecliptic. The Sun crosses it twice a year. The two points of intersection of the path of the Sun with the equatorial plane are the Spring (vernal) and Autumn equinoxes (*nox* is

Latin for night). They are the times at which the lengths of day and night are everywhere equal (see Fig. 1.1). The planets are Mercury, Venus, Earth, Mars, Jupiter, Saturn, Neptune, Uranus and Pluto, in order of increasing distance from the Sun (see Fig. 1-2). (With the discovery of many trans-Neptunian objects which look like smaller versions of Pluto, the designation of Pluto as a planet has come into question.) The first analyses of planetary motions used geometry, not physics.

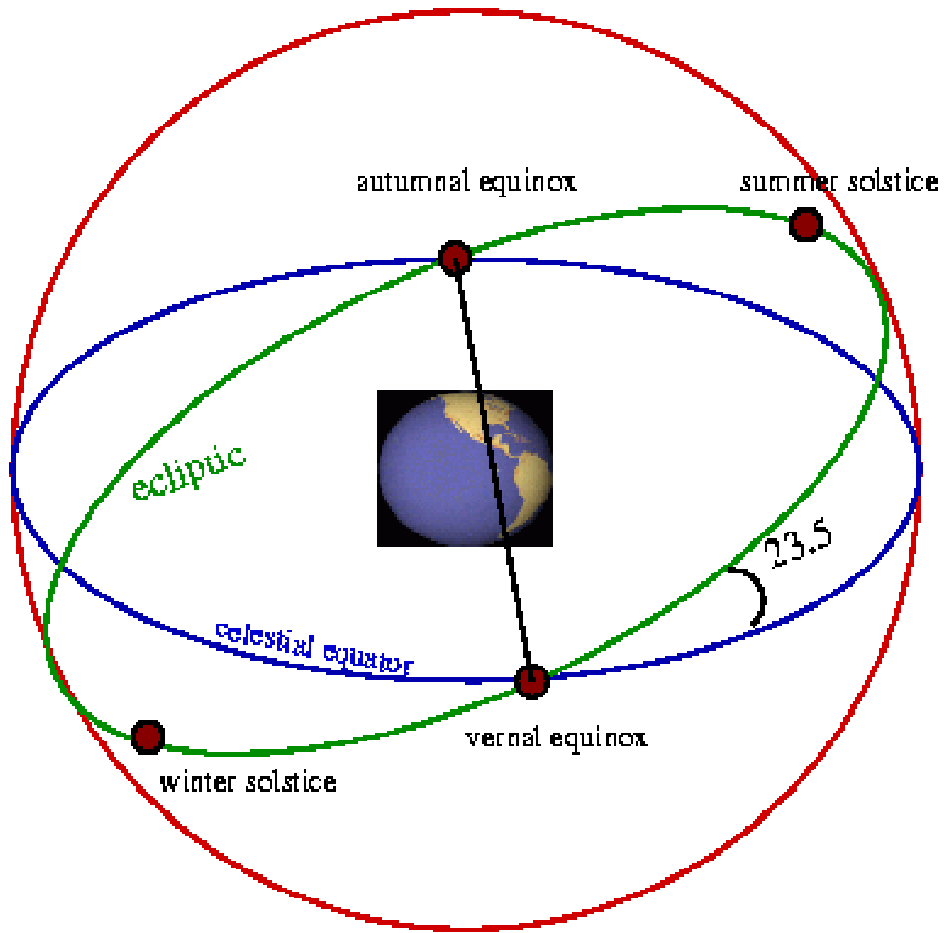
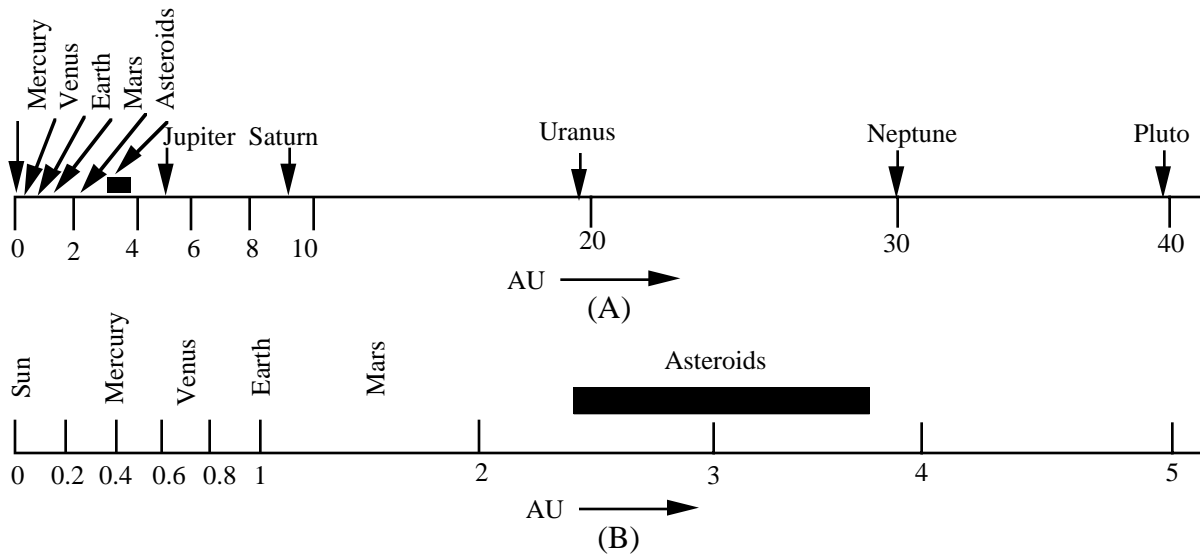


Fig. 1-1



Relative orbital distances of the planets. (A) The distance scale in astronomical units from the Sun to Pluto. (B) An expanded scale shows the inner Solar System

Fig 1-2

1.2.1 Geometry

Copernicus (early 16th century) put forward the simplifying suggestion that the planets are in circular orbits around the Sun. (We will see later that the orbits are ellipses). Planets are inferior or superior. Mercury and Venus are *inferior* planets because they lie closer to the Sun than the Earth. The others, lying further from the Sun are *superior* planets. The *elongation* of a planet is an angle, the angle measured at the Earth between the direction from Earth of the center of the Sun and the direction from Earth of the planet. *Conjunction* and *opposition* occur when the Earth, Sun and Planet are in a straight line. Conjunction occurs for the elongation

0° —it is *inferior conjunction* when the planet lies between the Earth and the Sun and *superior conjunction* when it lies on the opposite side of the Sun. When the planet is at an elongation of 180° it is at *opposition*. Opposition occurs only for superior planets. When the elongation is 90° it is called *quadrature* (see Fig. 1-3) for superior planets. For inferior planets, the maximum elongation is called the *greatest elongation*. At greatest elongation, the Earth-Planet-Sun angle is 90° .

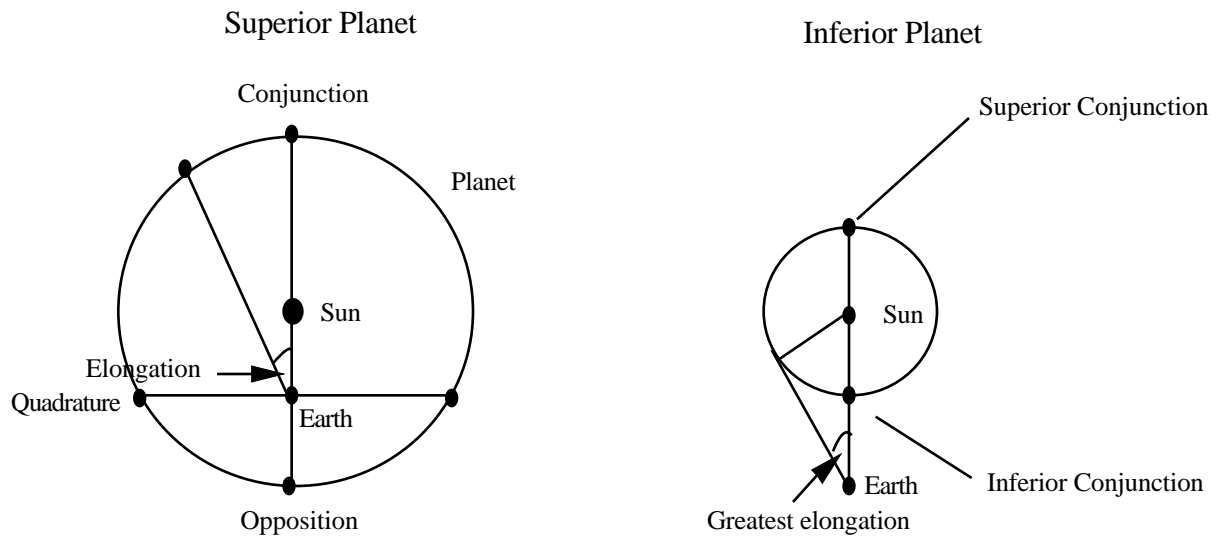


Fig. 1-3

Two periods are used to characterize the orbits of the planets. The synodic period S is the interval between successive oppositions or inferior conjunctions—it is the period measured from Earth. The sidereal period P is the actual time to make a complete orbit around the Sun. They are related, as Copernicus noted, by

$$\frac{1}{S} = \frac{1}{P} - \frac{1}{P_{\oplus}} \quad \text{for inferior planets}$$

$$\frac{1}{S} = \frac{1}{P_{\oplus}} - \frac{1}{P} \quad \text{for superior planets}$$

where P_{\oplus} is the sidereal period of the Earth

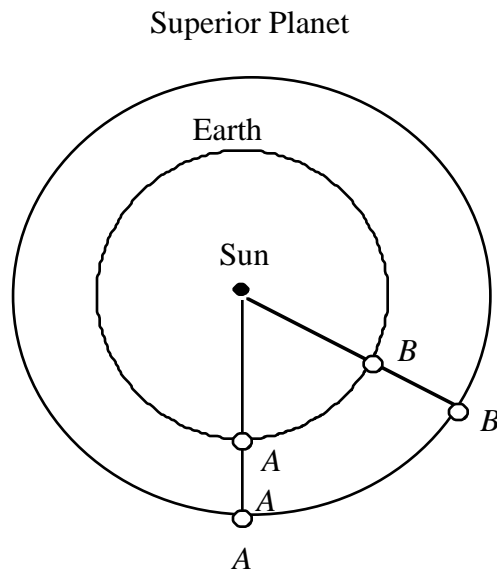


Fig. 1-4

Consider Fig. 1-4.

Opposition occurs at A . Suppose next opposition occurs at B . To get from A to B the planet takes time S and travels through the angle $(S/P) \times 2\pi$, $\frac{2\pi}{P}$ radians s^{-1} being its angular speed. In the same time, Earth has to travel one orbit taking a time P_{\oplus} so it has to travel from A to B in the remaining time $(S - P_{\oplus})$. In the time $(S - P_{\oplus})$ it travels through the angle $(S - P_{\oplus}) \frac{2\pi}{P_{\oplus}}$.

$$\frac{1}{S} = \frac{1}{P_{\oplus}} - \frac{1}{P} .$$

For superior planets, $P > P_{\oplus}$.

For inferior planets, it is the other way around.

We may define an angular velocity as the change in angle $\delta\theta$ in a time δt

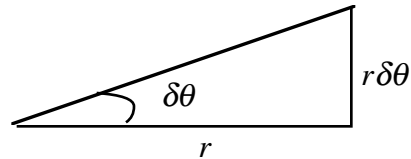
$$\omega = \frac{d\theta}{dt} .$$

$$S \times \frac{2\pi}{P} = (S - P_{\oplus}) \frac{2\pi}{P_{\oplus}}$$

For circular motion, ω is constant. If P is the period for a complete revolution in which θ changes by 2π radians and P is in seconds

$$\omega = \frac{2\pi}{P} \text{ rad s}^{-1} .$$

If r is the radius of the circular orbit, the velocity is $\frac{rd\theta}{dt} = r\omega$,



perpendicular to r .

We may express the relationship between periods in terms of angular velocities.

$$\omega_{\oplus} = \frac{2\pi}{P_{\oplus}}, \quad \omega = \frac{2\pi}{P}$$

$$\begin{aligned} \text{Then } \frac{2\pi}{S} = |\omega'| &= |\omega_{\oplus} - \omega| \\ &= \left| \frac{2\pi}{P_{\oplus}} - \frac{2\pi}{P} \right| \end{aligned}$$

or simply note that the difference in the angle traversed in time S by the planet, ωS , and the Earth, $\omega_{\oplus} S$, is 2π , i.e. $(\omega_{\oplus} - \omega)S = 2\pi$, as it should since S is the period as measured from Earth.

Copernicus also determined the relative distances of the planets from the Sun, though not the absolute distance. At maximum or greatest elongation, for an

inferior planet,

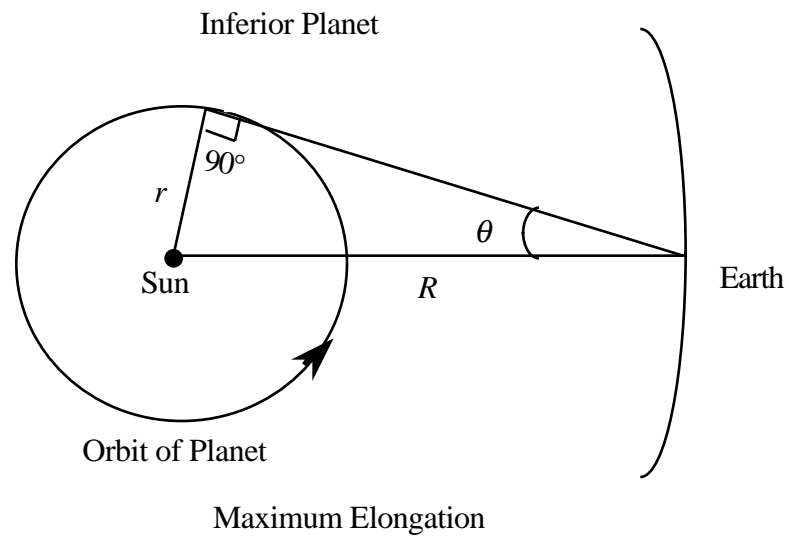


Fig. 1-5

If r is the radius of the orbit of the planet,

$r = R \sin \theta$ relates the Sun-Planet distance r to the Sun-Earth distance R ,

R , the distance of the Earth from the Sun, defines the astronomical unit of distance, $R = 1 \text{ AU}$.

The Copernicus method for a superior planet uses observations of opposition (so only one per orbit)

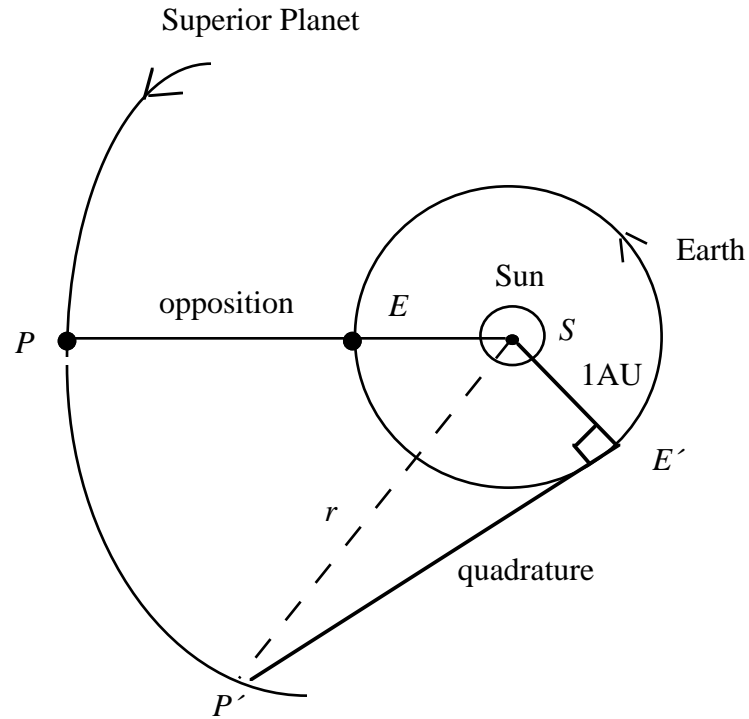


Fig 1-6

Observe at opposition P and at quadrature P' (angle between S and P' , $\hat{S}E'P' = 90^\circ$). $SE' = 1\text{AU}$. We know the time for the planet to move from P to P' and the Earth to move from E to E' . We know the angular speeds, so we know the angles $P\hat{S}P'$ and $E\hat{S}E'$. By subtracting we obtain the angle $P'\hat{S}E'$. Then $r = 1\text{AU}/\cos(P'\hat{S}E')$.

Kepler used a more elaborate geometry, applicable at all points of the orbit

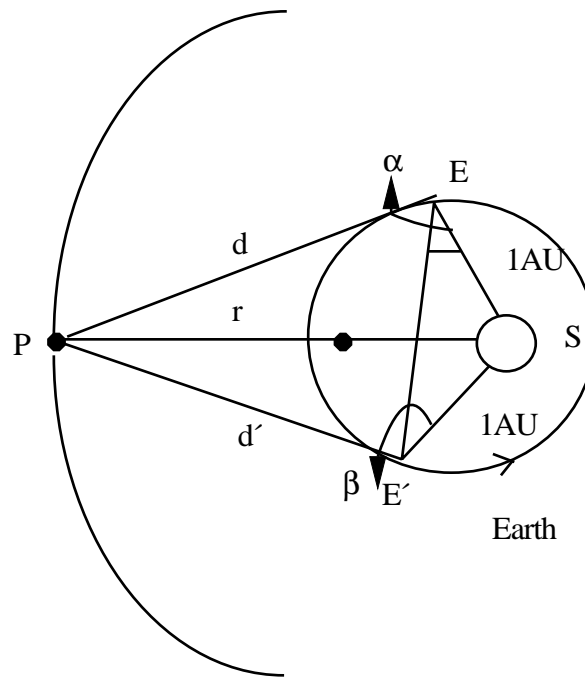


Fig 1-7

Planet P returns to chosen original position after one sidereal period and Earth moves from E to E' . Elongations α and β are measured. We know Earth's sidereal period so we know $\widehat{ESE'}$ and EE' in AU. Triangle is isosceles so we know angles $\widehat{SE'E}$ and $\widehat{SEE'}$. By subtraction, we obtain $\widehat{PE'E}$ and $\widehat{PEE'}$. Then sine rule

$$\frac{\sin \hat{a}}{a} = \frac{\sin \hat{b}}{b} = \frac{\sin \hat{c}}{c}$$

applied to triangle $\widehat{PEE'}$ gives

$$\frac{\sin(\widehat{PE'E})}{d} = \frac{\sin(\widehat{PEE'})}{d'} = \frac{\sin(\widehat{E'PE})}{EE'}$$

All these angles are known, so we derive d and d' . From either d or d' use

$$r^2 = d^2 + (1\text{AU})^2 - 2d \cos \alpha$$

or

$$r^2 = d'^2 + (1\text{AU})^2 - 2d' \cos \beta$$

Here are the measurements of P and of the average distance a from the Sun:

Measurements of P

Planet	P (days)	a (AU)
Mercury	87.969	0.387099
Venus	224.701	0.723332
Earth	365.256	1.000000
Mars	686.980	1.523691
Jupiter	4332.589	5.202803
Saturn	10759.22	9.53884
Uranus	30685.4	19.1819
Neptune	60189	30.0578
Pluto	90465	39.44

To determine 1 AU in some known unit, km, say, we need an absolute distance in both AU and km.

1.2.2 Parallax

In the case of planets, parallax is the apparent change in position of an object measured from different points of the Earth.

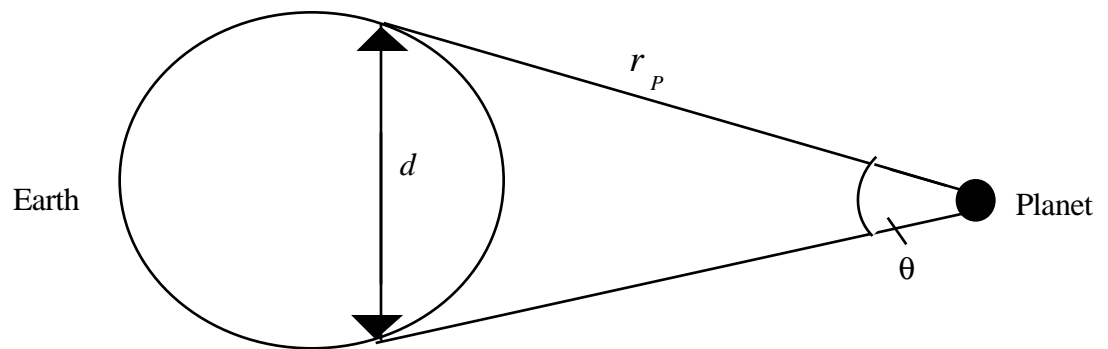


Fig. 1-8

Then $d/r_p = \sin \theta \sim \theta$, with θ in radians. d is called the baseline.

Knowing d in km, say, and r_p in AU, we may obtain AU in km from a measurement of θ . It was found that $1 \text{ AU} = 149\,597\,870 \pm 1 \text{ km}$. (the orbits are actually ellipses, and the Earth-Sun distance is not a constant. 1 AU is defined as the semi-major axis of the Earth's orbit which is equal to the average Earth-Sun distance) (cf. Chapter 4).

Knowing r_p , we can now determine the orbital velocities of the planets.

For the Earth,

$$r_E = 1.496 \times 10^8 \text{ km}$$

$$\omega = 2\pi \text{ rad yr}^{-1}$$

$$= 1.99 \times 10^{-7} \text{ rad s}^{-1}$$

$$v = r_E \omega = 2\pi \times 1.496 \times 10^8 \text{ km yr}^{-1}$$

$$= 9.40 \times 10^8 \text{ km yr}^{-1}$$

$$= 29.8 \text{ km s}^{-1} .$$

$$(1 \text{ yr} = 3.156 \times 10^7 \text{ s})$$

For stars, parallax is determined from different points of the Earth's orbit. The parallax angle is defined as half the apparent angular displacement of the star as seen from the Earth in half of one complete orbit.

Make observations of the positions of fixed stars from opposite points of the Earth's orbit.

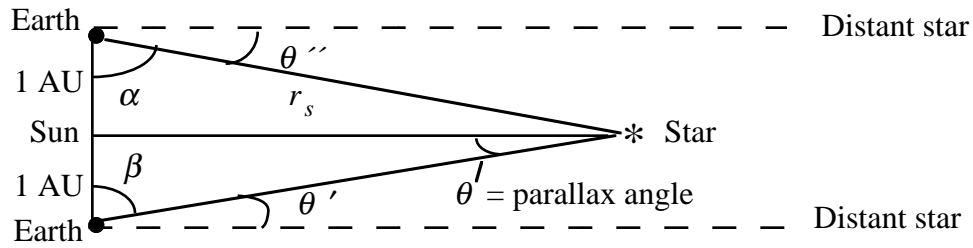


Fig. 1-9

$$\alpha + \beta + 2\theta = \pi$$

$$\pi - (\beta + \theta') = \alpha + \theta''$$

$$\text{Hence } 2\theta = (\theta'' + \theta')$$

In practice, choose observation times so that $\theta' = \theta''$. Then

$$r_s = \frac{1 \text{ AU}}{\tan \theta} \sim \frac{1 \text{ AU}}{\theta}$$

A distance unit, characteristic of stellar distances, is the parsec (abbreviated pc). A parsec is the distance r_s corresponding to a parallax angle of 1 arc sec for a baseline of 1 AU. With angle measured in radians and distance in AU,

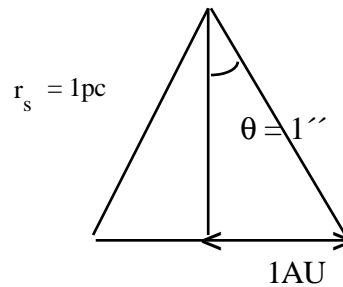
$$\frac{r_s}{\text{AU}} = \frac{\text{rad}}{\theta} .$$

Converting to angles measured in arcseconds, using 2π radians = $360 \times 60 \times 60$ arcsec,

$$r_s = \frac{360 \times 60 \times 60}{2\pi} \frac{1}{\theta} \text{ AU}$$

$$= \frac{206265}{\theta} \text{ AU} .$$

By definition $\theta = 1$ arcsec corresponds to $r = 1\text{pc}$.



So $1\text{pc} = 206265 \text{ AU}$
 $= 3.086 \times 10^{13} \text{ km}$
 $= 3.086 \times 10^{18} \text{ cm}.$

Another unit in common use is the light year—the distance travelled by light in one year.

$$1 \text{ ly} = 3 \times 10^{10} \text{ cm s}^{-1} \times 3.156 \times 10^7 \text{ s} = 9.467 \times 10^{17} \text{ cm}$$

$$\text{so } 1 \text{ pc} = 3.26 \text{ lyr.}$$

(Milky Way galaxy has a radius of about 7 kpc).

1.2.3 Transit of Venus

Venus is observed, though rarely, to transit the Sun. (The next two transits will occur in June 2004 and 2012.) The transits of Venus (the passages of Venus across the face of the Sun) were used to establish a distance scale for the Solar System. Venus appears as a small black dot moving across the face of the sun (see Fig. 1.10).

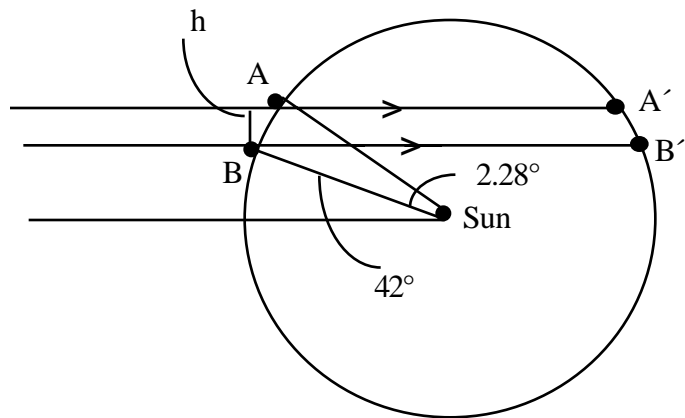
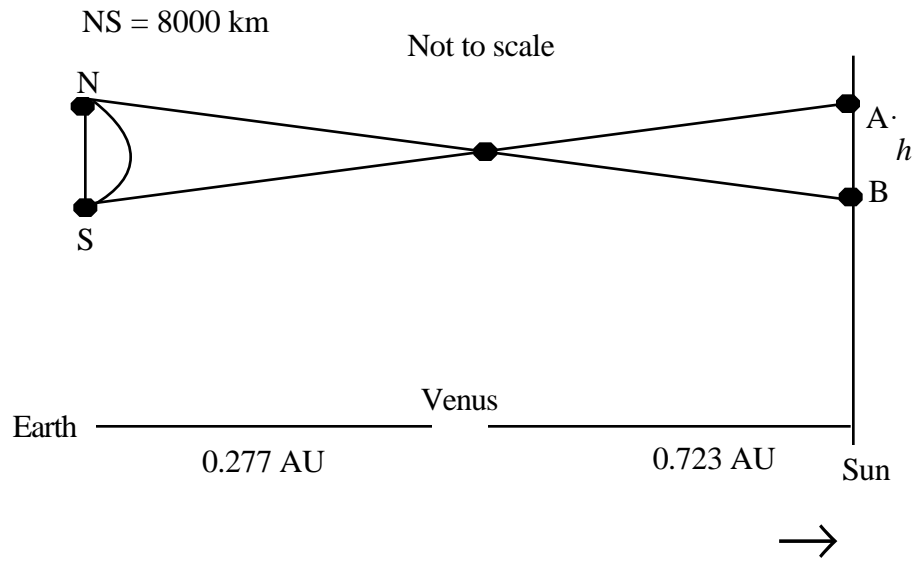


Fig. 1-10

The dot is actually moving in an arc on the surface of the Sun. When it is viewed from different locations on Earth, it appears to move in straight lines on slightly different tracks at different latitudes of the Sun (Figs 1-10 and 1-11).

We can determine an absolute distance scale from the difference in apparent latitudes 2.28° of the transits. We can also measure the different crossing times for which we can get the angles, knowing the Sun's angular size.

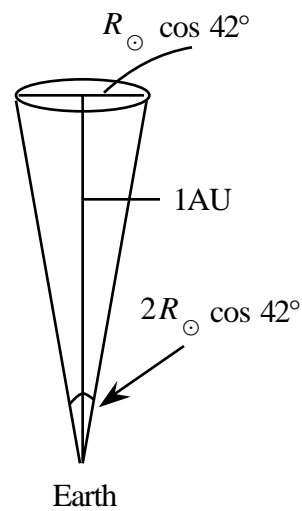
The angular distance travelled is

$$BB' = 2 R_{\odot} \cos 42^\circ$$

with R_{\odot} measured in AU.

Crossing rate is the relative angular velocity of the Earth and Venus.

$$\begin{aligned} \omega_r &= \frac{2\pi}{S} = \frac{2\pi}{1.60 \text{ years}} \\ &= 4.49 \times 10^{-4} \text{ rad/hour} . \end{aligned}$$



Measured crossing time at 49° latitude is 15.4 hours so with R_\odot in AU

$$\frac{BB' = 2R_\odot \cos 42^\circ}{4.49 \times 10^{-4}} = 15.4$$

and angular size of the Sun is

$$R_\odot = \frac{15.4 \times 4.49 \times 10^{-4}}{2 \times 0.743} \text{ rad}$$

$$= 4.65 \times 10^{-3} \text{ rad} = 16' .$$

The measured crossing time along arc AA' was 14.8 hours.

$$AA' = \frac{2R_\odot \cos (42^\circ + \Delta)}{4.49 \times 10^{-4}} = 14.8$$

which yields $\Delta=2.28^\circ$. Thus Northern observers see Venus crossing at 42° solar latitude, and Southern observers see Venus crossing more to the North of the Sun at latitude 44.28° . The difference in angle is $2.28^\circ = 0.0398$ radians. Because θ is small, the arc AB can be replaced by the line AB and AB is perpendicular to AO . See Fig. 1.10. Then $h = AB \cos 42^\circ$. But $(h/NS) = 0.72/0.28$ since $NS = 8000$ km, so $h = 20,571$ km and $AB = 27681$ km. With $\theta = 2.28^\circ = 0.0398$ radians, R_\odot

$$= 6.96 \times 10^5 \text{ km.}$$

Knowing the angular size of the Sun—32 arcminutes = $32'$, we obtain the Sun-Earth distance $1 \text{ AU} = R_{\odot}/\tan 16' = (6.96 \times 10^5/0.00465) = 1.496 \times 10^8 \text{ km.}$

1.2.4 Luminosity

Luminosity is the amount of energy emitted in unit time by a radiant object, whose radiant *flux* is defined as the amount of energy crossing a unit surface in unit time. Given that energy is conserved (no loss of energy by absorption), the flux diminishes with distance as $1/r^2$. (The total surface area increases as $4\pi r^2$ so the unit surface area expressed as a fraction of the total diminishes as $1/r^2$).

To reduce the flux from an object—the Sun—use a pinhole of area A , say.

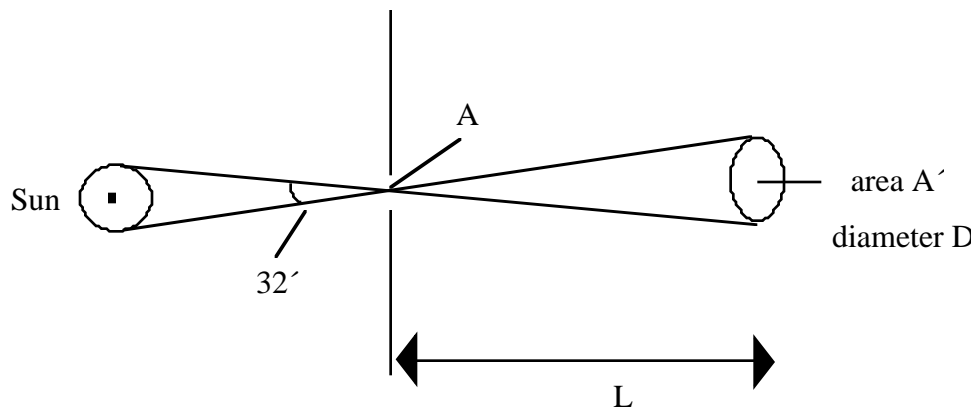


Fig 1-12

$\frac{A}{L}$ is the *focal ratio*)

All light collected by A is spread out over area A'.

Now the angle subtended by A' is also 32' so

$$A' = \frac{\pi D^2}{4} = \frac{\pi}{4} \left(L \frac{32}{60} \frac{2\pi}{360} \right)^2,$$

changing 32' to radians. Brightness (flux) is diminished by

$$\frac{A'}{A} = 6.0 \times 10^{-5} \frac{L^2}{A}.$$

Original unit of luminosity was a candle and the candle power L_{\odot} of the Sun was determined to be 2.8×10^{27} candles (Huygens 1650).

We now use Watts or ergs per second or Joules ($1 \text{ W} = 10^7 \text{ ergs s}^{-1} = 1 \text{ Joule s}^{-1}$). The solar luminosity L_{\odot} is given by the *solar constant*, the energy flux F_{\odot} received at Earth from the Sun,

$$F_{\odot} = \frac{L_{\odot}}{4\pi r^2}, \quad r = 1\text{AU}.$$

The measured solar constant is 1370 Wm^{-2} and the solar luminosity is

$$L_{\odot} = 4\pi r^2 \times 1370 \text{ W/m}^2$$

$$= 4\pi (1\text{AU})^2 1370 \text{ W}$$

With $1\text{AU} = 1.496 \times 10^{11} \text{ m}$,

$$L_{\odot} = 3.85 \times 10^{33} \text{ erg s}^{-1}.$$

1.2.5 Circular motion

$$r = \text{constant}, r^2 = \text{constant}$$

$$\text{Now } r^2 = \mathbf{r} \cdot \mathbf{r}$$

$$\text{so } \frac{d}{dt}(\mathbf{r} \cdot \mathbf{r}) = 2\mathbf{r} \cdot \frac{d\mathbf{r}}{dt} = 0.$$

$$\frac{d\mathbf{r}}{dt} = \text{velocity } \mathbf{v} \text{ so } \mathbf{r} \cdot \mathbf{v} = 0$$

$\therefore \mathbf{v}$ is perpendicular to \mathbf{r}

$$\text{Consider } \frac{d}{dt} \mathbf{v}^2 = \frac{d}{dt} v^2 = 0$$

$$\frac{d}{dt} \mathbf{v}^2 = 2\mathbf{v} \cdot \frac{d\mathbf{v}}{dt} = 2\mathbf{v} \cdot \mathbf{a}$$

\mathbf{a} is acceleration.

\mathbf{a} is perpendicular to \mathbf{v} and so is along the radius. Now, to find $|\mathbf{a}|$ consider that

$$\frac{d}{dt}(\mathbf{r} \cdot \mathbf{v}) = 0 = \frac{d\mathbf{r}}{dt} \cdot \mathbf{v} + \mathbf{r} \cdot \frac{d\mathbf{v}}{dt} = v^2 + \mathbf{r} \cdot \mathbf{a}$$

So $\mathbf{a} = -\frac{v^2}{r} \hat{\mathbf{r}}$ $\hat{\mathbf{r}}$ is unit vector along r .

\mathbf{a} is the *centripetal acceleration* along the radius to the center and $m\mathbf{a}$ is the corresponding *centripetal force*. Its magnitude is $\frac{mv^2}{r}$ or $m\omega^2 r$ since $v = \omega r$.