Astronomy 45

Introduction to Astrophysics

Problem Set 8 Due Friday November 16, 2001

1. a) Show that the time average of the potential energy over one orbital period τ can be reduced to the form

$$\langle V \rangle = \frac{-1}{\tau} \int_0^\tau \left(\frac{GM\mu}{r} \right) dt = A \int_0^{2\pi} r d\theta ,$$

where A is a constant factor.

b) Evaluate the integral from part (a), using the fact that

$$\int_0^{2\pi} \frac{dx}{1 + a \cos x} = \frac{2\pi}{\sqrt{1-a^2}} .$$

Show that the resulting expression is equal to <2E> , where E is the total orbital energy.

c) If <V> = 2E, show that the virial theorem is satisfied.

2. A particle starts from rest at an infinite distance from a star of mass M and radius R. The kinetic energy of the particle will be converted to heat and light when it impacts the surface. If the star is a white dwarf with $M = 1\ M_\odot$ and $R = 7 \times 10^8$ cm, what is the energy released by 1 g of infalling matter? What fraction is it of the rest mass (mc^2)? If the star were a neutron star with $M = 1.4\ M_\odot$ and R = 10 km, what would be the energy released? There are

sources which emit *X*-rays with a luminosity of 10^{37} ergs s^{-1}. If they are produced by material pulled from a companion star onto the surface of a neutron star, how much mass must be transferred per second, measured in solar masses?

3. What is the height of the tide on the Moon caused by the gravitational force exerted by the Earth? The radius of the Moon is 1738 km. The mass ratio of the Earth to the moon is 81 and the Earth-Moon distance is 380,000 km.

4. Two stars are bound together in a binary system. The primary star is losing mass to the secondary star at a rate $\dot{M}_1$. The radius of the secondary star is R_2 and its mass is M_2. If the infalling matter is assumed to be falling from rest at an infinite distance (as in qn.2) and the accreted matter is the sole source of heating of the secondary, show that the surface temperature of the secondary is given by

$$T_2 = \left(\frac{G M_2 \dot{M}_1}{4\pi\sigma R_2^3} \right)^{1/4} .$$