A brief discussion on solid angle

The defintion of a line element in spherical coordinate is

$$d\mathbf{r} = \hat{r}dr + \hat{\theta}rd\theta + \hat{\phi}r\sin\theta d\phi \tag{1}$$

For a constant radius surface, the area element is

$$dA = r(r\sin\theta)d\theta d\phi = r^2\sin\theta d\theta d\phi = r^2 d\Omega \tag{2}$$

where, the solid angle $d\Omega$ is defined as $d\Omega = \sin\theta d\theta d\phi$ and is the cone at an angle θ and area dA. The angle is measured from the direction perpendicular to the surface, so that $dA\cos\theta$ is the area dA projected onto a plane perpendicular to the direction in which, for instance, the radiation is traveling.

The volume element subtended by the solid angle $d\Omega$ is

$$dV = r^2 dr \sin\theta d\theta d\phi \tag{3}$$

The light that we receive from a star comes from stellar atmoshpere, due to thermonuclear reactions in the star's center. Much can be gained by studying the light from stars. For instance, the photon wavelength and intensity give clues as to the thermal conditions in a star. In order that we may explain such properties, it is crucial to lean how light travels through the gaseous layers that make up a star.

If $\epsilon_\lambda d\lambda$ is the amount of energy that the rays with wavelength between λ and $\lambda + d\lambda$ passing through the area dA at an angle θ into a cone of solid angle $d\Omega$, carry in a time interval dt, the specific intensity of the light rays is defined as

$$I\lambda = \frac{\epsilon_\lambda d\lambda}{d\lambda dA\cos\theta dt d\Omega} \tag{4}$$

The specific intensity has the units of energy per unit time per unit area per unit wavelength per steradians. For a body radiating as a blackbody, the specific intensity is the Planck function, i. e. $I_\lambda = B_\lambda$.

In your notes, it is shown how in the absence of extinction (loss of radiation due to collision or absorption), the intensity remains constant along any ray traveling through space. In general, the specific intensity varies with direction, so a mean intesity of the radiation is found as

$$J_\lambda = \frac{1}{4\pi} \int I_\lambda d\Omega \tag{5}$$

If the radition is isotropic (same intensity in all directions), such as for a blackbody, $J_\lambda = I_\lambda$.