Astronomy 45
Introduction to Astrophysics

Final Examination — 2:15 PM Tuesday May 23
Spring 2000 — Sever 113

Part A consists of questions requiring short answers and counts for 40% of the total score. Part B consists of proofs and problems and counts for 60%. You can use calculators. The following data may be useful:

1 AU = 1.496×10^8 km

G = 6.67×10^{-8} cm^3 s^{-2} g^{-1}

$\sigma = 5.67 \times 10^{-5}$ ergs cm^{-2}s^{-1} K^{-4}

$a = 7.56 \times 10^{-15}$ ergs cm^{-3} K^{-4}

$L_\odot = 3.90 \times 10^{33}$ ergs s^{-1}

$R_\odot = 6.96 \times 10^5$ km

Solar constant = 1370 W/m^2

$M_\odot = 1.99 \times 10^{33}$ g

1 $W = 10^7$ ergs s^{-1}

$$B_\nu(T) = \frac{2h\nu^3}{c^2} \frac{1}{e^{h\nu/kT} - 1}$$

$k = 1.38 \times 10^{-16}$ ergs K^{-1}

$h = 6.63 \times 10^{-27}$ ergs sec

Part A

Short Answers

1. Define the maximum elongation of an inferior planet.
2. Define a parsec. Given that 1 AU = 1.496×10^8 km, what is a parsec in km?
3. Write down Kepler's Third Law. Show that the orbital velocity of a planet decreases as its orbital radius increases.

4. Explain the difference between apparent and absolute stellar magnitudes.
5. Write down the equation of an ellipse with eccentricity ε and latus rectum r_o in plane polar coordinates.
6. What is the proper motion of a star?
7. Describe a method for determining the radius of a star given its luminosity *L* and surface flux *F*.
8. What is the color temperature of a luminous object?
9. What are the characteristic temperature ranges for O stars and M stars?
10. Which are more abundant: elements with even or elements with odd atomic mass numbers and why?
11. What are the differences between Pop I and Pop II stars?
12. The wavelength at which the Planck spectrum peaks is proportional to what power of the temperature?
13. What is the recombination era?
14. Why did Einstein introduce the cosmological constant?
15. What is coronal equilibrium?

Part B

Problems

1. An object is in a circular orbit around the Earth. It has a period of 1.5 years. What are the possible values for the radius of its orbit?

2. Show that the moment of inertia of a uniform spherical spinning body of radius R and mass M is given by $I = \frac{2}{5}MR^2$. A star of solar dimensions with radius $R_\odot$ collapses to form a neutron star of radius 16 km. No mass is lost in the collapse and the star is initially spinning at a rate of 12 rotations per year. Calculate the spin period after the collapse in seconds.

3. The albedo of Venus is 0.77 (albedo is the ratio of the reflected and incident radiation fluxes) and its distance from the Sun is 0.72 AU. If Venus radiates as a blackbody, what would be its temperature (ignore the effects of its atmosphere)? What is the difference in apparent magnitudes of the Sun and Venus? The radius of Venus is 6052 km.

4. Show that the equation of hydrostatic equilibrium that relates the pressure P to the density ρ of a star is
$$\frac{dP}{dr} = -\frac{G\rho(r)M(r)}{r^2}$$
where r is the distance from the center and $M(r)$ is the mass inside r. The radius of the star is R and its total mass is M. Derive a formula for the pressure as a function of r in terms of M and R for the case where the density ρ is constant. The degeneracy pressure for a non-relativistic white dwarf is proportional to $\rho^{5/3}$. Show that MR^3 is a constant. What happens in the relativistic case?

5. Give a brief account of the history of the early Universe through to the recombination era. What reactions occurred in the period of nucleosynthesis and what elements were created?

6. An HII region has a radius of 10 pc and a proton density of 300 cm^{-3}. The rate coefficient for recombination of electrons and protons is 2.8×10^{-13} cm^3 s^{-1}. How many ionizing ultraviolet photons per second are absorbed by the gas if recombinations and photoionizations balance? The number of ionizing photons from a star of mass M is given approximately by

$$N = 4 \times 10^{40} (M/M_{\odot})^{5.7} \text{ s}^{-1}.$$

What is the mass of the star? What is its spectral class?