Astronomy 45
Introduction to Astrophysics

Final Examination — 2:15 Friday May 15
Spring 1998 — Sever 214

Part A consists of questions requiring short answers and counts for 40% of the total score. Part B consists of proofs and problems and counts for 60%. You can use calculators. The following data may be useful:

$1\ \mathrm{AU} = 1.496 \times 10^{8}\ \mathrm{km}$

$G = 6.67 \times 10^{-8}\ \mathrm{cm^3\ s^{-2}\ g^{-1}}$

$\sigma = 5.67 \times 10^{-5}\ \mathrm{ergs\ cm^{-2} s^{-1}\ K^{-4}}$

$L_{\odot} = 3.90 \times 10^{33}\ \mathrm{ergs\ \ s^{-1}}$

$R_{\odot} = 6.96 \times 10^{5}\ \mathrm{km}$

$1\ W = 10^{7}\ \mathrm{ergs\ s^{-1}}$

$$B_{\nu}(T) = \frac{2h\nu^2}{c^3}\ \frac{1}{e^{h\nu/kT} - 1}$$

Part A.
Short Answers

1. a) Write down the relationship between the synodic and sidereal periods of an object that is in orbit around the Sun.

   b) Write down Kepler's Third Law.

   c) An object is observed from the Earth to have a synodic period of 2 years. What is the ratio of the two possible values of the semi-major axis of the object's orbit?

2. The energy of a particle moving in an elliptical orbit determines the semi-major axis. What property of the orbit is determined by the angular momentum?

3. Briefly describe what observations you would make to determine stellar distances in terms of km.

4. What is the relationship between apparent stellar magnitudes and absolute stellar magnitudes?

5. In the classification scheme U, B, V, which has the longest wavelengths and which has the greatest energy per photon?

6. What parameters are related in the Hertzsprung-Russell diagram? What is meant by a “main sequence star”?

7. What are the Roche lobes in a binary system?

8. What determines the Roche limit for a satellite of a planet?

9. Explain why the moon always offers the same face to the Earth.

10. What is the energy source for stars of solar mass?

11. What is the Doppler effect?

12. How does electron degeneracy pressure arise?

13. What is a polytropic gas and how does the polytropic index n relate to the adiabatic index γ?

14. What is the difference between emission nebulae and reflection nebulae?

15. What are the mean molecular weights of a neutral gas of hydrogen and a fully-ionized plasma of protons and electrons?

Part B.

Problems

1. A spacecraft is launched form Earth to Saturn on the orbit requiring the least energy. The Sun-Saturn distance is 9.54 AU. How long a time does the journey take? Assume motion takes place in the gravitational field of the Sun only.

2. A meteorite approaches the Earth from an infinite distance with an initial velocity v_i moving along a straight line that in the absence of gravity would pass the Earth at a perpendicular distance b. The meteorite just misses the Earth's surface. If the initial velocity v_i is equal to the velocity of a particle on the Earth's surface, what is b in units of the Earth's radius?

3. What would be the relationships between the Einstein absorption, stimulated and spontaneous emission coefficients if photons were fermions in which case the occupation number would be $2/(e^{h\nu/kT} + 1)$?

4. If the position vector **r** is measured from the center of mass of the Earth and the Moon, the local tidal potential may be written.

$$\sigma(\mathbf{r}) = \frac{-GM_1}{a} - \frac{1}{2}\frac{Ga^2}{R^3}\left(3M_2 \cos^2\theta + M_1\right)$$

where M_1 is the mass of the Earth, M_2 is the mass of the Moon, R is the Earth-Moon distance, a is the distance of the point at **r** from the center of the Earth and θ is the angle between the Earth-Moon line and the vector connecting the point of **r** to the center of the Earth. What is the height of the tides on the Moon induced by the Earth? The lunar radius is 1738 km, the Earth-Moon mass ratio, M_1/M_2 , is 81 and the Earth-Moon distance is 380,000 km.

5. Prove the virial theorem and confirm that for a gravitational field the mean kinetic energy T and the mean potential energy V satisfy $V = -2T$. What is the relationship between V and T for a potential varying as the n^{th} power of the distance $V(r) = -kr^n$?

6. A star has a constant core density ρ_o out to a radius r_o. Beyond r_o out to the stellar radius R, the density $\rho(r)$ varies with radius r as $\rho_o\ (r_o^2/r^2)$. Calculate the mass $M(r)$ interior to r and obtain the stellar mass $M(R)$ in terms of ρ_o , r_o and R. Write down the equation of hydrostatic equilibrium and calculate the pressure $P(r)$ as a function of radius r. [The pressure is zero at the radius R and continuous across the core boundary.]