

**Astronomy 45**  
**Introduction to Astrophysics**

Final Examination  
Spring 1999

2:15 Friday May 28  
Emerson 108

Part A consists of questions requiring short answers and counts for 40% of the total score. Part B consists of proofs and problems and counts for 60%. You can use calculators. The following data may be useful:

$$1 \text{ AU} = 1.496 \times 10^8 \text{ km}$$

$$G = 6.67 \times 10^{-8} \text{ cm}^3 \text{ s}^{-2} \text{ g}^{-1}$$

$$\sigma = 5.67 \times 10^{-5} \text{ ergs cm}^{-2} \text{ s}^{-1} \text{ K}^{-4}$$

$$a = 7.56 \times 10^{-15} \text{ ergs cm}^{-3} \text{ K}^{-4}$$

$$L_{\odot} = 3.90 \times 10^{33} \text{ ergs s}^{-1}$$

$$R_{\odot} = 6.96 \times 10^5 \text{ km}$$

$$M_{\odot} = 1.99 \times 10^{33} \text{ g}$$

$$1 \text{ W} = 10^7 \text{ ergs s}^{-1}$$

$$B_{\nu}(T) = \frac{2h\nu^2}{c^3} \frac{1}{e^{h\nu/kT} - 1}$$

$$k = 1.38 \times 10^{-16} \text{ ergs K}^{-1}$$

$$h = 6.63 \times 10^{-27} \text{ ergs sec}$$

$$c = 3 \times 10^{10} \text{ cm s}^{-1}$$

**Part A**

## Short Answers

1. What are the sidereal and synodic periods and how are they related?
2. What law do you need to show that the orbital velocities of the planets decrease with distance from the Sun?
3. What is the parallax angle and what is the definition of 1 parsec?
4. Define apparent visual magnitude. What is the bolometric correction?
5. What is the red shift? What does it tell us about the Universe?
6. Write the Planck function  $B_{\lambda}(T)$  as a function of wavelength  $\lambda$ . What form does it take at long wavelengths?
7. Express the luminosity  $L$  of a black body of radius  $R$  in terms of the surface flux  $F$ .
8. What physical parameter determines the spectral class of a star? What is the Hertzsprung-Russell diagram?
9. What is Kepler's second law?
10. What is a spectroscopic binary?
11.  $^{15}\text{O}$  decays to  $^{15}\text{N}$  plus two other particles. What are these particles?
12. What causes the tides?
13. What is the form of the equation of state of a polytrope of index  $n$ ?
14. What is the Stromgren radius?

**Part B**

## Problems

1. A satellite is in orbit around the Earth. The greatest and least altitudes of the satellite above the Earth's surface are 1200 km and 400 km. Calculate the period of one orbit. The mass of the Earth is  $5.97 \times 10^{27}$  g.
2. Show that the motion of two bodies of masses  $M_1$  and  $M_2$  moving under the influence of their mutual gravitational attraction can be reduced to the notion of a single fictitious particle of reduced mass  $\mu$  at the center of mass. If particle 1 is the Earth with mass  $5.97 \times 10^{27}$  g and particle 2 is the Moon with mass  $7.37 \times 10^{25}$  g, what is the average radius of the orbit of the Earth about the center of mass?
3. A star is represented by a spherical blackbody of radius  $5.16 \times 10^6$  km and temperature 28,000 K. It is located at a distance of 180 pc from the Earth. Calculate the radiative flux at the surface of the star and the luminosity of the star. What is the flux at the Earth? At what wavelength is the intensity a maximum?
4. A star has a mass of  $10^{35}$  g and a luminosity of  $4 \times 10^{39}$  erg  $s^{-1}$ . Assume the star is made initially of hydrogen which is converted to helium by nuclear energy. The mass of a proton is 1.0078 AMU and the mass of an  $\alpha$ -particle is 4.0026 AMU. 1 AMU =  $1.66 \times 10^{-27}$  kg. For how long can the star radiate before the supply of hydrogen is used up?
5. A star has a mass  $M$  and a radius  $R$ . Its density  $\rho(r)$  is a constant,  $\rho_c$ , independent of the distance  $r$  from the center. Write down the equation of hydrostatic equilibrium and integrate it to obtain the pressure as a function of  $r$ . Use the perfect gas law to obtain an expression for the temperature  $T(r)$  in terms of  $M$  and  $R$ .
6. A flux of photons of  $10^4$   $cm^{-2}$   $s^{-1}$  at a specific frequency  $\nu$  is incident on a gas of neutral hydrogen atoms of density  $10$   $cm^{-3}$ . The photoionization cross section at  $\nu$  is  $10^{-18}$   $cm^2$ . The recombination coefficient of protons and electrons is  $10^{-12}$   $cm^3$   $s^{-1}$ . Calculate the equilibrium fractional ionization  $n(H^+)/n(H)$  of protons to neutral hydrogen atoms, given that  $n(H^+)/n(H)$  is small compared to unity. If the flux is switched off at a time  $t_0$ , how does the electron density decay with time  $t$ ?