# AY 145: HW 9

*Due Wednesday, April 27th*

1. **Age of the Universe**

   (a) Find the lifetime of a closed universe (expressed as a multiple of the Hubble time, $t_{\rm H}$) as a function of the density parameter, $\Omega_0$.

   (b) Suppose the universe is closed and flat. From the facts that the Sun is about 4.5 billion years old and you live in an expanding universe, what limit could you put on the present value of the density parameter, $\Omega_0$? Consider the cases of $h = 0.5$ and $h = 1$.

2. **Cosmological Distances** Write a computer program (any language, any machine — this is for you, not me!) to calculate the proper, luminosity and angular diameter distances as a function of redshift, for any $H_0$, $\Omega_M$ and $\Omega_\Lambda$. Plot these for H=70 km/s/Mpc and for $\Omega_M$ = 0.0, 0.3 and 1.0, and $\Omega_\Lambda$ = 0.0 and 0.7, as appropriate, for z= 0 to 6.

   You will need to numerically integrate the equations for distance in a Lambda Cosmology:

$$d_P \;=\; R(t)\int_0^{r_1} \frac{dr_1}{(1-kr_1^2)^{1/2}}$$

   which can be written, in a matter and $\Lambda$ dominated model (no radiation), for the Luminosity Distance as

$$d_P \;=\; \frac{c}{H_0}|\Omega_K|^{-1/2} sinn\left\{|\Omega_K|^{1/2}\int_0^{z} \frac{dz'}{((1+z')^2(1+\Omega_M z') - z'(2+z')\Omega_\Lambda)^{1/2}}\right\}$$

   where $\Omega_K \;=\; 1-\Omega_M-\Omega_\Lambda$, $sinn(x) = sin(x), (x)$, or $sinh(x)$ for k= 1, 0, and -1 respectively (and $\Omega_k$ has the *opposite sign* from k: $\Omega_k$ = -k/$H_0^2$), and the other terms are self evident. Also remember that $d_L = (1+z)d_P$ and $d_A = d_L(1+z)^{-2}$, where $d_P$ is the comoving or "proper" distance.