## Project 1

Assigned February 9, 2012
Due February 23, 2012
Assume a spherical atmosphere, us76.dat, us76.spline

1. Construct a layered atmosphere up to 50 km , with 5 km layers: Using the CurtisGodson approximation ${ }^{1}$ find the best representative pressure, $\mathrm{P}_{\text {eff }}$, for each layer and set the layer temperature ${ }^{2}$ and height $\left(\mathrm{T}_{\text {eff }}, \mathrm{Z}_{\text {eff }}\right)^{2}$ to the corresponding values from the US76 atmosphere.
2. Nadir geometry: For the following conditions:

- wavelengths of 350 and 1000 nm
- albedos of $0.02,0.2$, and 1.0 : Lambertian
- apparent solar zenith angles of 0,45 , and 80 degrees (take this as the angle where the solar beams hits the Earth's surface)
- satellite viewing angle of 0 degrees
- Rayleigh single scattering as both a source and an extinction term
(a) Calculate and plot the refractive path through the atmosphere (applying Snell's law ${ }^{3}$ at the layer boundaries) for the spherical atmosphere.
(b) Go to the plane-parallel approximation, and look at the three lowest layers (the "troposphere"). Considering the Rayleigh scattering ${ }^{4}$ as a loss to $\mathrm{I}_{0}$ and a source term, from scattering, in each of the three layers, determine the upwelling radiation at the satellite viewing angle as a fraction of $\mathrm{I}_{0}$ and plot the contribution seen by each layer to the total. (In other words, what fraction of the light exiting the atmosphere has seen each layer?) Ignore refraction in this part and ignore polarization induced by Rayleigh scattering (by the way, solar radiation is highly non-polarized). Include the source term from light backscattered from the surface, which undergoes extinction on the way up. Ignore higher order Rayleigh scattering source terms.

Do a separate calculation, lowest layer only, of the source term from $2 \pi$ steradians of surface backscattered light Rayleigh scattered by the lowest layer into the satellite field of view.
3. Limb scattering: For the following conditions ${ }^{5}$ :

- spherical atmosphere from (1); lowest 3 layers only
- wavelengths of 350 and 1000 nm
- albedo $=0.1$, Lambertian
- solar zenith angles of 0 and 45 degrees
- solar azimuth angle of 45 degrees (here I mean the azimuth angle with respect to the sun and the measurer, not the astronomical solar azimuth angle, i.e., $\left.\varphi-\varphi_{0}=45^{\circ}\right)$.
- Rayleigh single scattering as both a source and an extinction term
- ignore refraction
(a) Calculate and plot the radiation at the satellite as a fraction of $\mathrm{I}_{0}$ for viewing tangent to each layer's lower boundary. Treat the sun as a source of parallel rays and then consider the terms for scattering of incoming light, Lambertian reflection at the surface and a second scattering along the lines-of-sight (use the plane-parallel approximations for this, as developed in the last calculation in Part 2). The scattering terms may be approximated as scattering from the same physical conditions ( $\mathrm{P}_{\text {eff }}, \mathrm{T}_{\text {eff }}, \mathrm{Z}_{\text {eff }}$ ) as in Part 1, but with geometric factors appropriate to the problem.
(b) For measuring tangent to 10 km , and considering the sphericity of the atmosphere (but not refraction), what are the effective pressure and height, in terms of column density of molecules along the path for the $10-15 \mathrm{~km}$ layer?
${ }^{1}$ The Curtis-Godson approximation states that the effective pressure is given by $\bar{P}=\frac{\int P \rho d z}{\int \rho d z}$, where $\rho$ is the density (for the effective pressure of a species with fractional concentration $c$, it is given by $\bar{P}=\frac{\int P c \rho d z}{\int c \rho d z}$ ). It is exact for Lorentz (i.e., pressure) broadened absorption or emission lines in the limits of very strong and very weak, and deviates in between. See Houghton, Section 4.4, and problem 4.13 for details.
${ }^{2}$ For temperature interpolation, you may wish to use a cubic spline interpolation, in order to gain experience with it. See cspline.f90 on the website. Otherwise, linear interpolation is fine.
${ }^{3}$ Index of refraction, Snell's law: $n_{1} \sin \theta_{1}=n_{2} \sin \theta_{2}$, where the angles are perpendicular to the interface. For the index of refraction at 288.15 K and 1013.25 millibar (mb) use: $(n-1) \times 10^{8}=8060.77+\frac{2481070 .}{132.274-\lambda^{-2}}+\frac{17456.3}{39.32957-\lambda^{-2}}$, where $\lambda$ is the wavelength in micrometers (microns, $\mu \mathrm{m}$ ), and scale $(n-1)$ as density, $\propto \mathrm{P} / \mathrm{T}$ : This is equation 21 of Bodhaine, B.A., N.B. Wood, E.G. Dutton, and J.R. Slusser, On Rayleigh optical depth calculations, J. Atmos. Ocean. Tech. 16, 1854-1861, 1999. (on website)
${ }^{4}$ Rayleigh scattering is given by (Bodhaine, equation 29):
$Q_{R} \times 10^{28}=\frac{1.0455996-341.29061 \lambda^{-2}-0.90230850 \lambda^{2}}{1+0.0027059889 \lambda^{-2}-85.968563 \lambda^{2}}$, where $\lambda$ is the wavelength in $\mu \mathrm{m}$, and $Q$ is in $\mathrm{cm}^{2}$ (to be multiplied by the column density in $\mathrm{cm}^{-2}$ to determine the fraction of light scattered out of the beam). The scattering phase function is
$\Phi=\frac{3}{4}\left(1+\cos ^{2} \theta\right)$, normalized to $\int \Phi d \Omega=4 \pi$.
${ }^{5}$ The scattering angle $\Theta$ is determined from (cf. Goody and Yung, eq. 8.2) $\cos \Theta=\cos \theta \cos \theta_{0}+\sqrt{1-\cos ^{2} \theta} \sqrt{1-\cos ^{2} \theta_{0}} \cos \left(\varphi-\varphi_{0}\right)$, where $\theta_{0}$ and $\varphi_{0}$ are the solar zenith and azimuth angles and $\theta$ and $\varphi$ are the viewing zenith and azimuth angles.


## Project 2

Assigned March 22, 2012 due April 12, 2012
This project consists of calculation of realistic atmospheric spectra for several measurement geometries. Ignore refraction in all calculations.

1. Infrared solar absorption spectrum for HCl R 1 lines at $2923.732 \mathrm{~cm}^{-1}\left(\mathrm{H}^{37} \mathrm{Cl}\right)$ and $2925.897 \mathrm{~cm}^{-1}\left(\mathrm{H}^{35} \mathrm{Cl}\right)$. Using the plane-parallel atmosphere from before, calculate pure absorption spectra for SZA of $45^{\circ}$ for a spectral region covering these lines. Spectral resolution should be several times smaller than the Doppler line width. Line shapes are Voigt (voigt363.f90). Species included are $\mathrm{HCl}, \mathrm{H}_{2} \mathrm{O}, \mathrm{O}_{3}, \mathrm{CH}_{4}$, and $\mathrm{NO}_{2}$. Plot the result and then convolve with a typical instrument line width of $0.01 \mathrm{~cm}^{-1}$ (use a Gaussian of $0.01 \mathrm{~cm}^{-1} \mathrm{HW} 1 / \mathrm{e}$, either using voigt363.f90 or gauss.f90) and replot.
2. OH limb emission in far infrared. Calculate limb spectrum through entire atmosphere for tangent heights of $25,30,35,40,45 \mathrm{~km}$. Spectral region 118-119 $\mathrm{cm}^{-1}$. Spectral resolution $0.0002 \mathrm{~cm}^{-1}$ (approximately the Doppler width). Plot the result and then convolve with a typical instrument line width of $0.004 \mathrm{~cm}^{-1}$ (use a Gaussian of 0.004 $\left.\mathrm{cm}^{-1} \mathrm{HW} 1 / \mathrm{e}\right)$ and re-plot. Use the geometric paths and the $P_{\text {eff }}$ and $T_{\text {eff }}$ values from Project 1.
3. Satellite back-scattered radiation: Calculate the ratio $I / I_{0}$ for absorption of sunlight by ozone, $300-330 \mathrm{~nm}$. Using the plane-parallel atmosphere from before, calculate pure absorption spectra for SZA of $45^{\circ}$, viewing angle of $0^{\circ}$, albedo $=0.1$ (You may, if you prefer, construct a new layered atmosphere with 2 tropospheric layers and 2 stratospheric layers). Use wavelength grid of cross sections. Plot results. Then, add Rayleigh scattering as loss and source terms, re-calculate, re-plot.

Or, alternatively:
Calculate the back-scattered spectrum for CO in the overtone region $4150-4350 \mathrm{~cm}^{-1}$. Use bottom 25 km only, and include $\mathrm{CO}, \mathrm{H}_{2} \mathrm{O}$, and $\mathrm{CH}_{4}$, obtained from HITRAN on the web. CO profile can come from B\&S Appendix D or another source (e.g., GEOSChem modelers at Harvard). Use albedo $=0.03$ (appropriate to land). Ignore Rayleigh scattering (why?) What spectral resolution appears to be necessary to successfully measure CO? What are the advantages and disadvantages of measuring the overtone versus the vibrational fundamental?

Intensities and widths in spectral line files given at 296 K . Approximate $Q(T)$ as $\propto T^{1}$ (linear molecules) or $\propto T^{1.5}$ (nonlinear molecules) in order to calculate $S(T)$ for each line in each layer.

## Files:

brasseur-solomon-profiles.dat concentration profiles from B\&S oh-hitran.project OH line list extracted from HITRAN
hcl-hitran.project HCl line list extracted from HITRAN
o3bp.project Ozone cross sections with temperature dependence
voigt363.f90 Voigt subroutine subprogram
voigt363-function.f90 Voigt function subprogram
gauss.f90 alternate (to Voigt) Gaussian convolution for instrument width hitran-molparam.txt HITRAN isotope codes and molecular weights

