3. Elements of math and physics

Several concepts and formulas that are generally useful in atmospheric radiative transfer are gathered here for later use.

3.1 Units for light wavelengths and frequencies

Note that spectroscopy is somewhat stuck at *cgs* units (centimeter-gram-second) as opposed to the more modern and preferred *MKS* (meter-kilogram-second), and still sometimes uses Ångstroms for wavelengths in the visible and ultraviolet (10Å = 1 nanometer). Remembering that $E = h\nu = hc\sigma = hc/\lambda,$ where h is Planck's constant (6.6260693×10^{-27} erg s) and σ is used for *wavenumbers* (cm^{-1}):

Visible

← wavelength (λ) frequency (ν) →

						1000	100	10	nm
			1000	100	10	1	0.1		μm
		1	10	100	1000	10^4	10^5		cm^{-1}
300	3000								MHz
		30	300	3000					GHz
			0.3	3	30				THz

|← **HITRAN** →| → *eV, MeV, TeV*

Table 1.1 Wavelength and frequency ranges

nm are nanometers, μm are micrometers (or "microns"), cm^{-1}, as before, are wavenumbers (not "inverse centimeters" or "reciprocal centimeters"), and the last three rows are megahertz, gigahertz, and terahertz. The distinction between the use of frequency versus wavelength is often due to instrument technique and/or spectral range: radiofrequency (RF), microwave, heterodyne techniques $\Rightarrow$ frequency; Fourier transform spectrometer (FTS) $\Rightarrow$ frequency (usually cm^{-1}), since their spectra are linear with energy. Dispersive instruments (grating/prism spectrometers) $\Rightarrow$ wavelength, since their spectra are linear with wavelength (approximately so in the case of prisms, more exactly so for gratings).

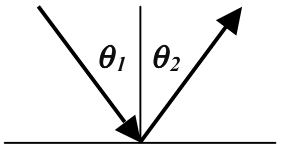
To emphasize, the relationships $E = h\nu = hc\sigma = hc/\lambda$ tell how to *convert among units*. Thus, a frequency of 200 cm^{-1} corresponds to a frequency of 5.995849×10^{12} Hz (5.995849 THz, 5.995849×10^{3} GHz, 5.995849×10^{6} MHz), and a wavelength of 0.005 cm (50 μm, 50,000 nm, 500,000 Å).

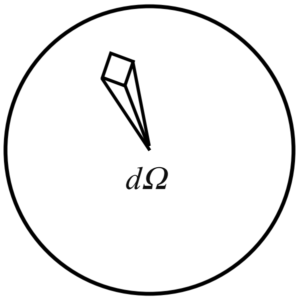
3.2 Lambertian reflectance and emission

A *Lambertian* radiation source has reflectance and emission proportional to $\cos\theta$, where θ is the angle normal to the surface. Lambertian emission is an approximation to real situations, but often a quite good one. The $\cos\theta$ dependence means that the observed emission is independent of the viewing angle with respect to the normal to the emitting surface. Reflected light when there is reflection, *i.e.*, for a surface that is not a blackbody, is also proportional to $\cos\theta$, and therefore independent of the viewing angle.

3.3 The bi-directional reflectance distribution function, *BRDF*

Normal reflectance:





The bi-directional reflectance distribution function is the generalized reflectance from a surface. It is, in general dependent upon the input angle normal to the reflecting surface, θ_1, the output angle normal to the surface, θ_2, and the azimuthal angle between them, φ. It is, in general, also dependent upon the polarization state of the input light. *BRDF* (θ_1, θ_2, φ) is often approximated by simpler forms. For example, for Lambertian reflectance, $R \propto \cos\theta_2$, so that $BRDF = k\cos\theta_2$, where k is a constant scaling the integrated reflectance.

3.4 Optical elements

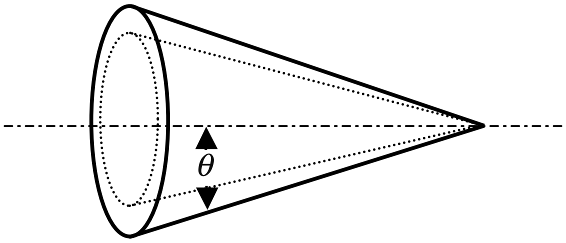
The following optical elements are of general use in applications of atmospheric spectroscopy and of associated instrumentation.

Cones

The surface area of a sphere of radius $r = 4\pi r^2$
The element of solid angle, $d\Omega = d\theta d\phi, \int_{sphere} d\theta d\phi = 4\pi$

The solid angle Ω subtended by a cone of half-angle θ is given by

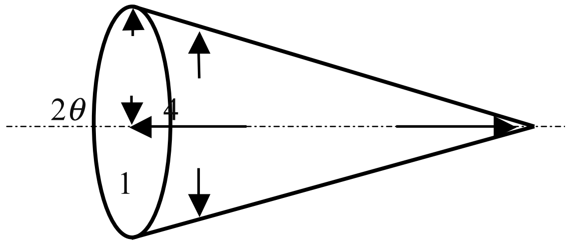
$$\Omega = 2\pi\int_0^{\theta} \sin\theta' d\theta' = 2\pi(1-\cos\theta).$$



For $\theta = 90^{\circ}$ (*i.e.*, a plane), $\Omega = 2\pi$ steradians (sr).
For θ small (and in radians), $\Omega \approx \pi\,\theta^2$.

Étendue:

Put a (small) hole, of area a, in the tip of the cone above. Then $É = a\,\Omega$. The étendue, the product of the entrance pupil and the subtended solid angle, is a constant of an optical system. It cannot be increased (although it can be effectively decreased, by *vignetting*, or blocking part of the optical signal). *a* and Ω may be altered in inverse proportion by an optical but not increased.



Ω large is described as a “fast” optical system; Ω small is described as a “slow” optical system, from the time required to take a photograph using a camera containing optical components with such angular acceptances.

the F-number or f-stop = $1/2\tan\theta$ (this shows an *f/2* system)
The f-stop is adjusted in photograph as a tradeoff between exposure
Time and image sharpness, with sharper images arising from slower settings.

It is important to realize that atmospheric (and astronomical and, indeed, laboratory spectroscopic) measurements are made with instruments having properties described by their étendues. It is often a convenient approximation (one we shall employ frequently) to describe spectroscopic problems as plane-parallel. The angular situation is always lurking underneath.

Diffraction limit

A uniformly illuminated circular aperture, our most usual first approximation to an optical system, presents a diffraction pattern, the Airy disk, with a large central bright region and concentric, successively smaller, bright rings. The dimensions of the Airy disk are determined by the aperture size. The overall diffraction pattern is the image of a point source on an extended detector. The Rayleigh criterion for resolution, for resolving two equally bright point sources, is that they are space apart angularly so that the center of one Airy disk corresponds to the first minimum of the other. This angular distance is the diffraction limit for a lens or mirror. It is given by $\theta = 1.22\ \lambda / d$, where θ is in radians λ is the wavelength of the light being detected and d the telescope diameter.

References

The Horiba Jobin Yvon Company has an excellent website giving a tutorial on the optics of spectroscopy: http://www.horiba.com/us/en/scientific/products/optics-tutorial/

Problems (assigned February 6, due February 18)

3.1 Construct a table showing wavelengths and frequencies (nm, μm, cm^{-1}, MHz, GHz, and THz) for: CO 1→0 and 2←0 band centers (2143.272 cm^{-1}; 4260.063 cm^{-1}); ClO MLS emission line (204.35 GHz); O_2 *A* band center (13120.909 cm^{-1}); CO_2 15 μm “greenhouse” band (667.380 cm^{-1}); O_3 TOMS “on” wavelength (317.35 nm).

3.2 Construct an example where one observes an extended source (*e.g.*, a cloud) with an instrument having a given étendue. Show that the étendue is the same for the cloud observing you.