5. Radiative transfer

5.1 Definitions

Radiative transfer is the process of energy transfer during the propagation of electromagnetic radiation. It includes the processes of extinction and emission. Extinction is completely described microscopically by absorption plus scattering:

$$\text{Extinction } (\varepsilon) = \text{absorption } (k) + \text{scattering } (m). \tag{5.1}$$

Extinction can be described by a dimensionless *optical thickness*, τ. τ is equal to the product of the cross section (for absorption and/or scattering) per molecule, Σ (in length^2), the length of absorption, l, and the concentration of absorbers/scatterers, c (in length^{-3}):

$$\tau = \Sigma \times l \times c. \tag{5.2}$$

Extinction of intensity I is then described by the Beer-Lambert law, $I = I_0 e^{-\tau}$. A differential increment, $d\tau = \Sigma \times c \times dl$.[1]

The *single scattering albedo (SSA)* is defined as

$$SSA = m / (k + m). \tag{5.3}$$

SSA = 1 is the condition for conservative scattering, scattering without absorption of radiation. *SSA* may be quite close to 1 for scattering of visible radiation by water droplets: Optical thickness for cloud scattering *(multiple scattering)* may be quite large ($\tau \sim 10$ as a global average for Earth but it can readily reach substantially larger values).

The description of scattering by atmospheric particles (aerosols) and gases and of emission by gas volumes, aerosols or surfaces further requires introduction of the scattering *Phase function* $\Phi(\theta, \phi, \hat{i})$, $\theta =$ the forward scattering angle, relative to the incident light, $\phi =$ the azimuthal angle, and $\hat{i} =$ the polarization of the scattered light for a given input polarization state. The phase function thus describes the angular distribution of light from a source, which may be either scattering of input light or radiation from imbedded sources.

Blackbody emission is isotropic, $\Phi = 1$. Most atmospheric scattering is not, as discussed in Chapter 11. The phase function is normalized to $\int_\Omega \Phi \, d\Omega = 4\pi$. This choice of normalization is made as light is scattered or emitted into 4π steradians of solid angle.

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[1] Optical depth is the cumulative optical thickness to a point into a medium (an atmosphere, for example) with extinction.

Thus our solid angle Ω, from the étendue (Section 3.4), will accept $\Omega/4\pi \times \Phi$ of the scattered radiation, or other source term.

5.2 The basic equation of radiative transfer

For incident intensity $I(\sigma)$, where σ is included to make frequency (wavenumber) explicit, the basic equation describing a scattering event is:

$$-dI(\sigma) = d\varepsilon I(\sigma) - dJ(\sigma), \tag{5.4}$$

where $d\varepsilon$ is the differential extinction and $dJ(\sigma)$ is the differential source term. A single scattering event may be pictured as in **Figure 5.1**.

The basic process for absorption plus emission, in an increment of radiative transfer with an optical thickness $\tau(\sigma)$ is illustrated (in the plane-parallel approximation, Chapter 3.4 and Problem 5.1) in **Figure 5.2**.

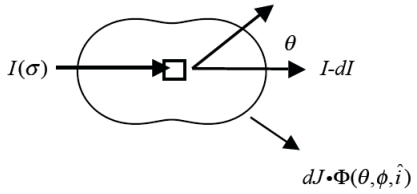

Figure 5.1. The geometry of a scattering event, including the differential source term scattered into the envelope of the scattering phase function, shown with strong forward and back scattering as in molecular (Rayleigh) scattering.

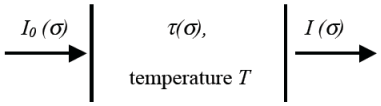

Figure 5.2. The geometry for absorption plus emission in the plane parallel approximation.

The outgoing radiation $I(\sigma)$ includes a Beer –Lambert extinction term plus a blackbody absorption term that is due to the integration of the source term in eq. 5.4:

$$I(\sigma) = I_0(\sigma)e^{-\tau(\sigma)} + B(\sigma,T)(1-e^{-\tau(\sigma)}), \tag{5.5}$$

where $B(\sigma,T)$ is the blackbody emission: $B(\sigma,T) = R_\sigma = \dfrac{c_1\sigma^3}{e^{c_2\sigma/T}-1}$.

Note that $\tau(\sigma)$ increments add (**Figure 5.3**)

$$I = I_0 e^{-(\tau_1(\sigma)+\tau_2(\sigma))} + B(\sigma,T)[1-e^{-(\tau_1(\sigma)+\tau_2(\sigma))}], \tag{5.6}$$

as long as regions are at the same temperature.

For "pure" emission (I_0 = 0, ~outer space in background), with a large $\tau(\sigma)$, the emission approaches, but never exceeds, that of a blackbody at the same temperature. For "pure" absorption

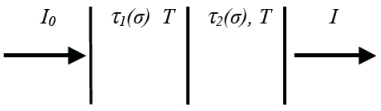

Figure 5.3. Absorption and emission by two successive isothermal atmospheric layers.

(approximated in the mid IR and higher for planetary atmospheres), large $\tau(\sigma) \Rightarrow I/I_0 = 0$. Looking at the pure cases:

"Pure" absorption: $I / I_0 = e^{-\tau(\sigma)}$

"Pure" emission: $\dfrac{I}{B(\sigma,\tau)} = 1 - e^{-\tau(\sigma)}$

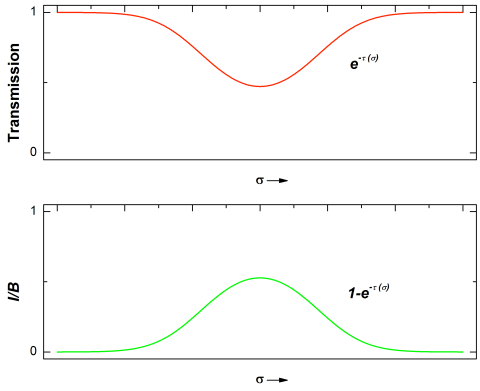

Figure 5.4. Illustration of the symmetry for absorption and emission by the same optical thickness spectrum. If the emission is normalized to the blackbody spectrum appropriate to its temperature it is exactly he inverse of the absorption spectrum.

When there are successive emitting layers at different temperatures, the final intensity is buffered toward the temperature of the final layer as illustrated in **Figure 5.5**.

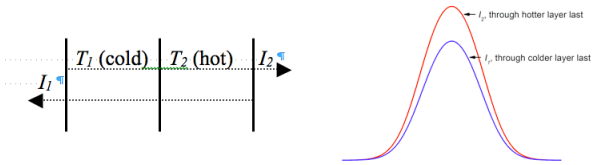

Figure 5.5. Buffering of the spectral intensity toward the nearest atmospheric layer.

Spectroscopy of planetary atmospheres may take advantage of absorbed, emitted, or scattered light, depending on the measurement geometry, the atmospheric constituents being measured and the available instrumentation. As measurements are often made from a location close to the object (*e.g.*, Earth) and far from the source of illumination (e.g., the Sun) the radiation received from the atmosphere can be a balance of a hotter distribution with smaller source solid angle and a cooler, more extended source. For near-Earth measurements the crossover point is in the infrared (see Problem 4.3), **Figure 5.6**. At longer wavelengths, in the infrared and microwave regions, measurements of emission spectra of the atmosphere are generally practical. At shorter wavelengths in the infrared, visible, and ultraviolet, measurements of surface or (especially in the visible and ultraviolet) atmospherically scattered light are normally used for measurements.

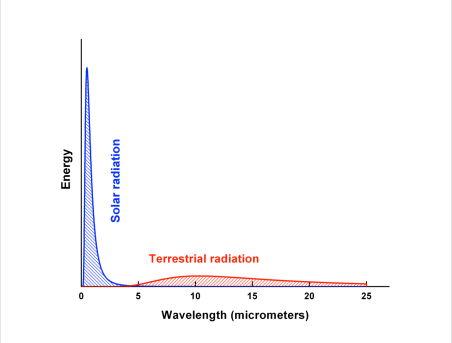

Figure 5.6. Contribution from terrestrial and solar radiation for Earth's atmosphere measurements from close to the Earth so that the field of view is dominated by Earth.

References

Goody, R.M., and Y.L. Yung, "Atmospheric Radiation: Theoretical Basis," 2nd Edition, Oxford University Press, New York, 1989. Chapter 2.1 is particularly relevant.

Chandrasekhar, S, "Radiative Transfer," Dover Publications, New York, 1960. The standard work on detailed radiative transfer.

Problems (assigned February 13, due February 25)

5.1 Demonstrate for Figure 5.2 how the geometric considerations cancel, permitting the plane-parallel approximation to be invoked.

5.2 Consider the problem of the measurement of atmospheric carbon monoxide (CO) by looking downward from an Earth satellite above the atmosphere. CO has a spectroscopic transition (the vibrational fundamental) at 4.7 micrometers ("microns," μm), and another at 2.3 μm (the first vibrational overtone). Assuming that the CO is distributed throughout the troposphere and that the Earth's surface is at nearly the same temperature as the lowest atmosphere, then (a) What sort of measurements may be made for each transition? (b) at what altitudes will measurements of each be sensitive to atmospheric CO? (c) Is there an advantage to measuring one or the other, or both, for determining atmospheric CO?