A Introduction to the light scattering in the atmosphere

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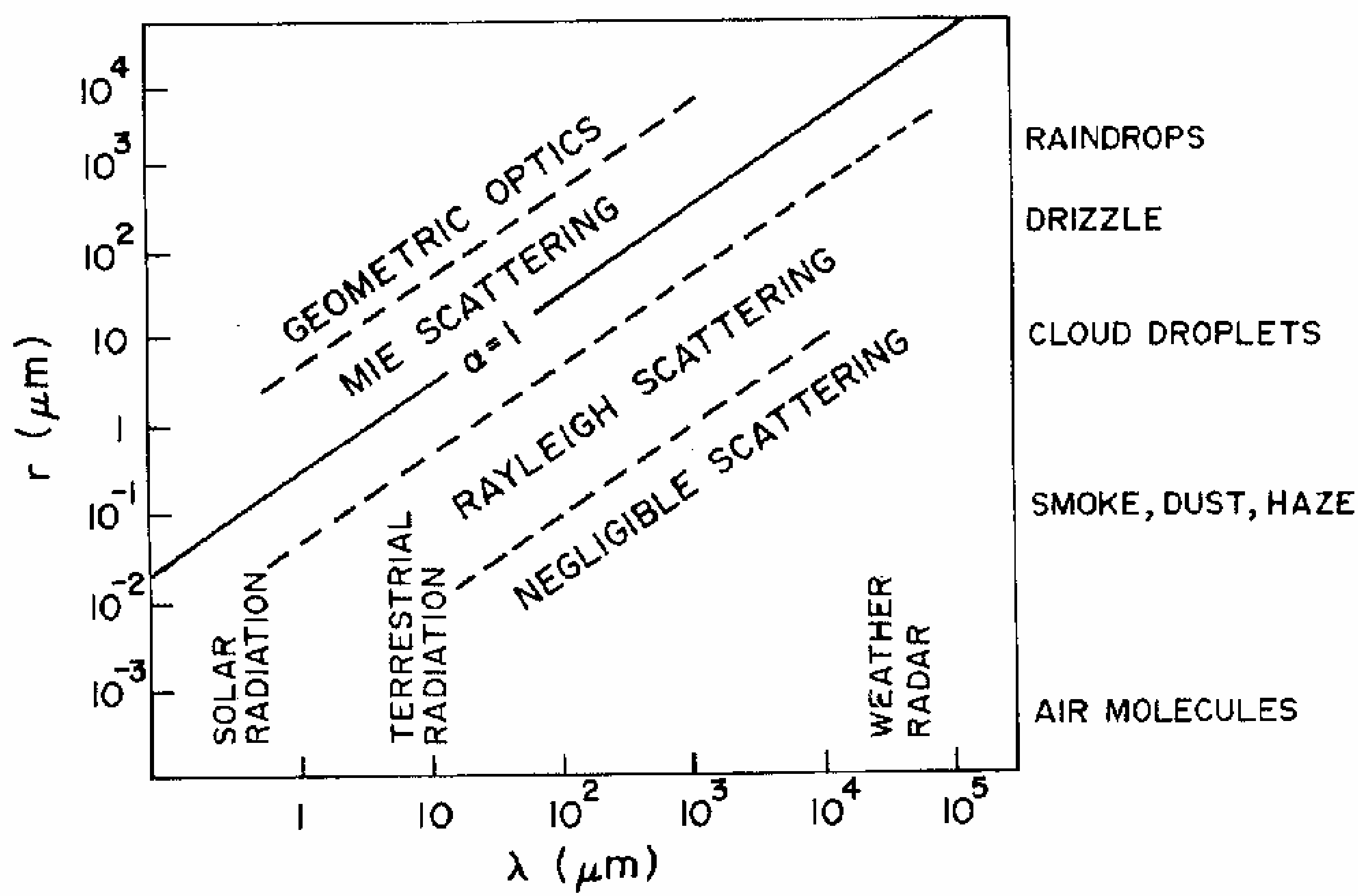
Outline

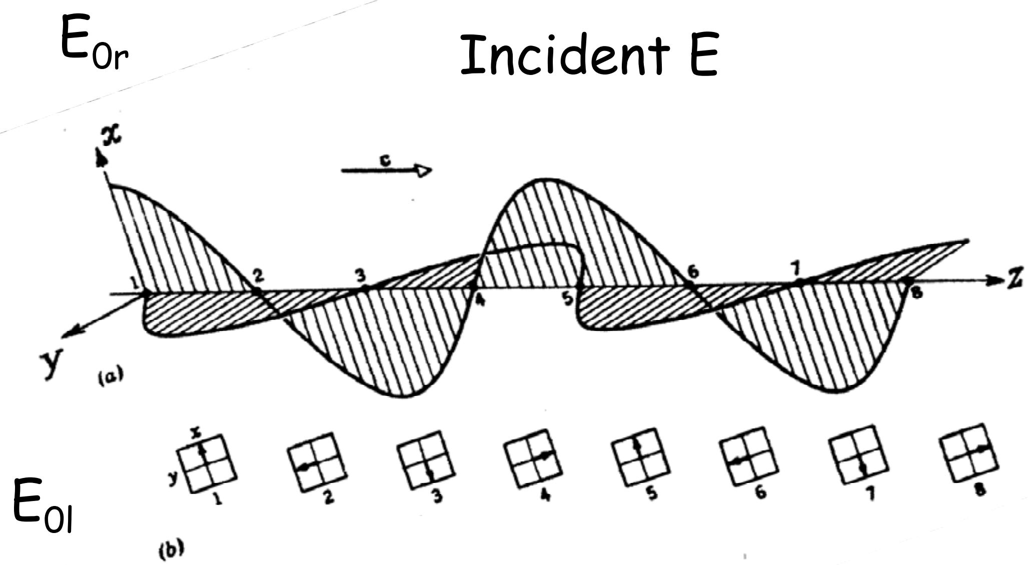
- Radiation from single and double dipoles
- Scattering by a single (multi-dipole) particle
  - Lorentz-Mie theory
  - Single scattering properties
  - Stokes parameter
- Single scattering properties of an ensemble particles
- Aerosol and water cloud droplet size distribution
- Scattering by a non-spherical particle
- Relevant literatures

Scattering regime

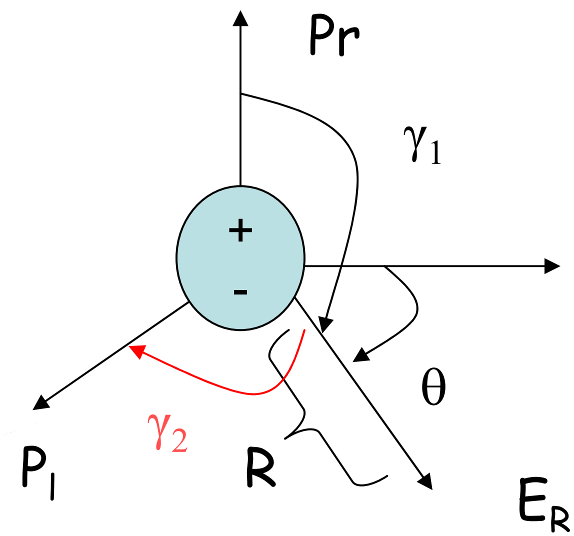


The scattering of solar and terrestrial radiation by atmospheric aerosols and clouds is mostly in the Mie scattering regime.

Radiation from a single dipole



Scattering by Dipole



$$E = \frac{1}{c^2}\frac{1}{r}\frac{\partial^2 P}{\partial t^2}\sin\gamma$$

Scattered E

$$P = P_0 e^{-ik(r-ct)}$$

Scattered dipole moment

$$P_0 = \alpha E_0$$

Induced dipole moment

Any polarization state can be represented by two linearly polarized fields superimposed in an orthogonal manner on one another

Rayleigh scattering

$$E_r = E_{0r}\frac{e^{-ik(R-ct)}}{R}k^2\alpha$$

Polarizability
$P=\alpha E_0$

Spherical wave form

$$E_\ell = E_{0\ell}\frac{e^{-ik(R-ct)}}{R}k^2\alpha\cos\Theta$$

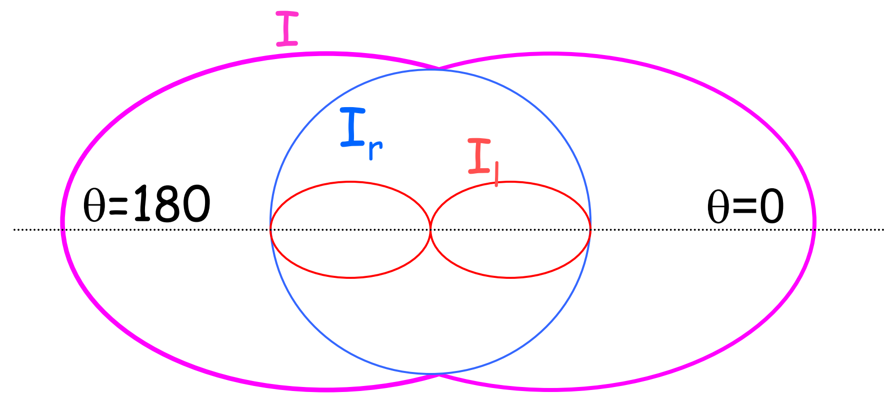
$I = |E|^2$

$$I_r = I_{0r}\frac{k^4|\alpha|^2}{R^2}$$

$$I_\ell = I_{0\ell}\frac{k^4|\alpha|^2}{R^2}\cos^2\Theta$$

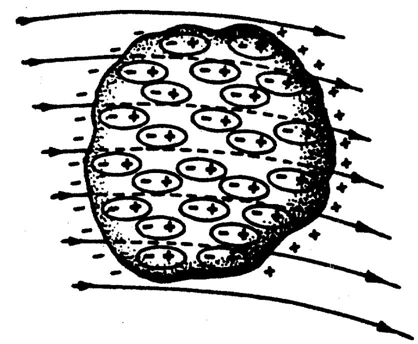
$(2\pi/\lambda)^4 \rightarrow \lambda^{-4}$

$$I = I_r + I_l = (I_{0r} + I_0\cos^2\theta)k^4\alpha/R^2$$

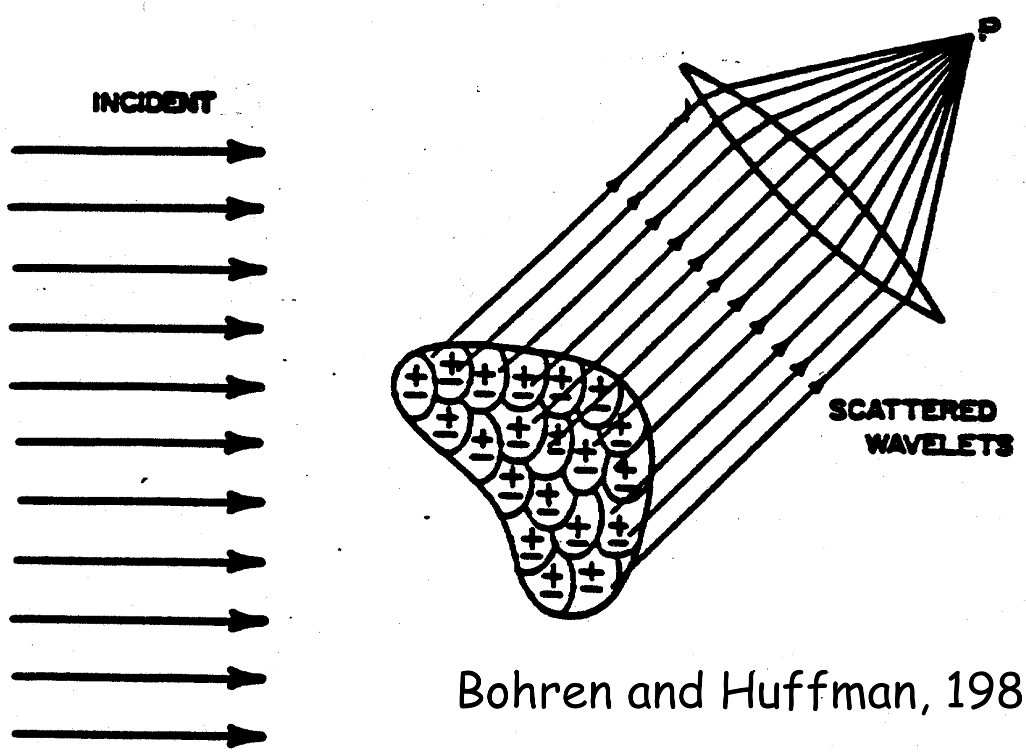


Phase function of Rayleigh scattering. g = 0

Like molecular absorption, the key property that determines scattering processes of a particle is whether or not the material readily forms dipoles.



Dipole oscillation generates EM



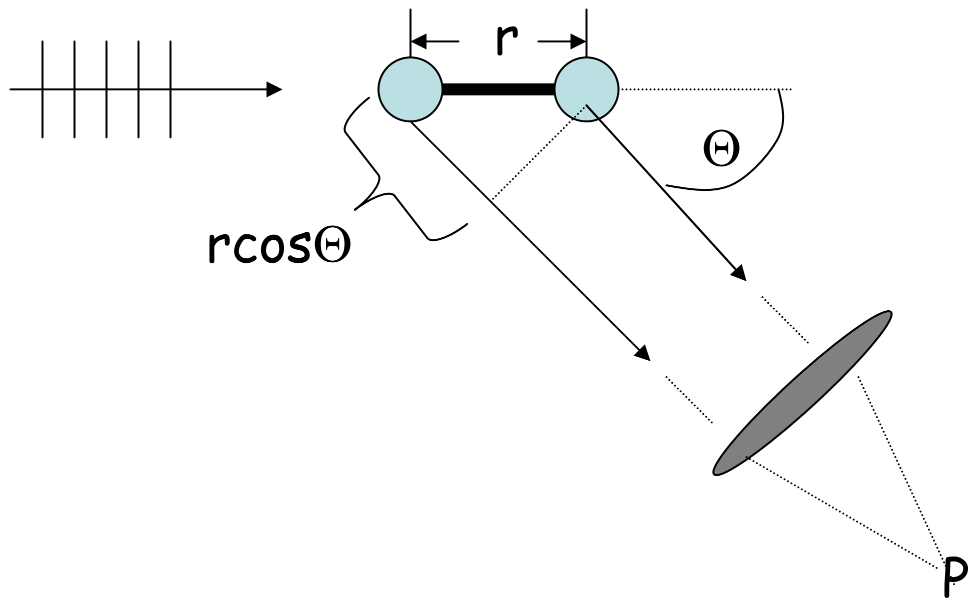
Bohren and Huffman, 1983

The analysis of particle scattering can be simplified by thinking that the scattered radiation is the composite contributions from many waves generated by oscillating dipoles that make up the particle.

The radiation scattered by a particle and observed at P results from sunperpositon of all wavelets scattered by the subparticle regions (dipoles)

A simple view of particle scattering

Radiation from a multiple dipole particle



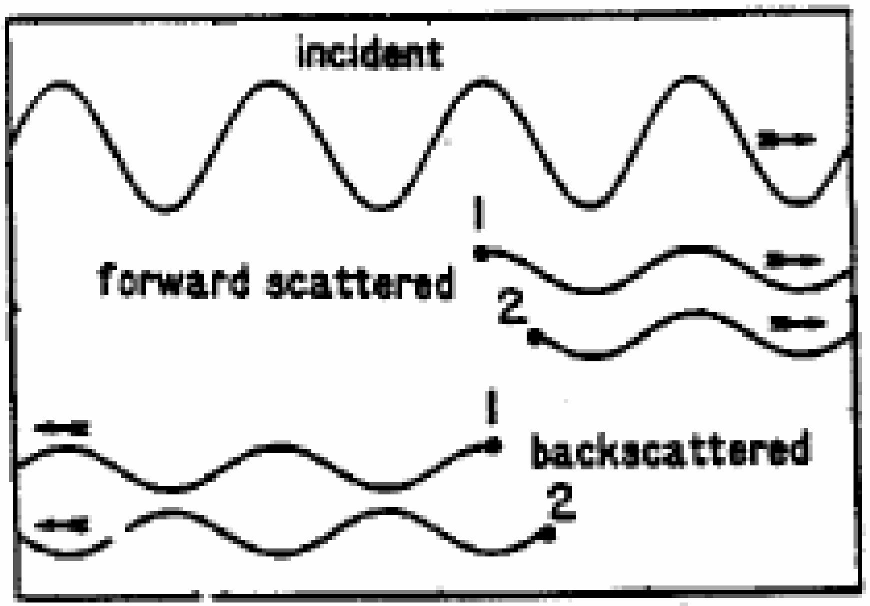
ignore dipole-dipole interactions

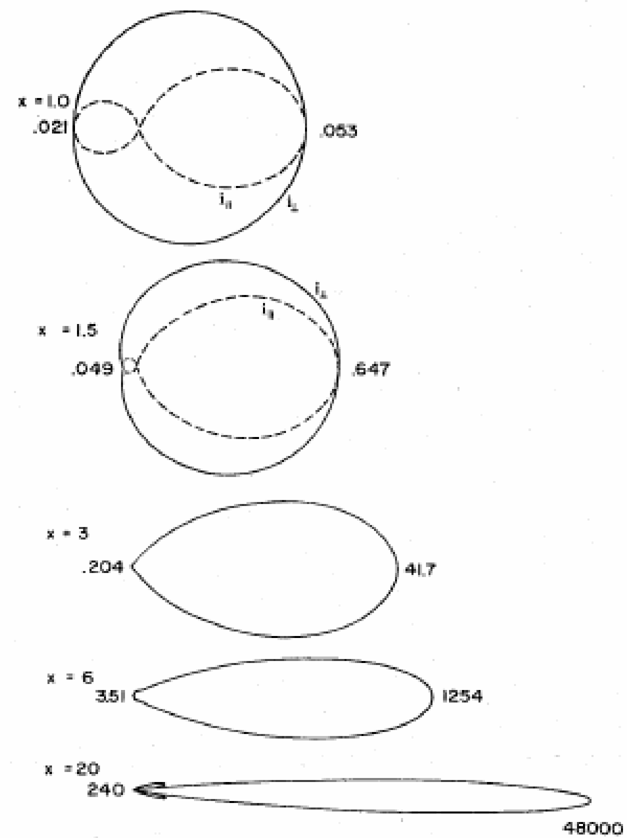
At P, the scattered field is composed on an EM field from both dipoles . The phase difference between waves E1 and E2 is proportional to the difference in path length:

$$\Delta\phi = 2\Pi r(1-\cos\theta)/\lambda$$

$$E_{1+2} = E_1 e^{i\phi} + E_2 e^{i(\phi+\Delta\phi)} = E_1^2 + E_2^2 + 2 E_1 E_2 \cos \Delta\phi$$

When $\theta = 0$, the E fields are always reinforced.





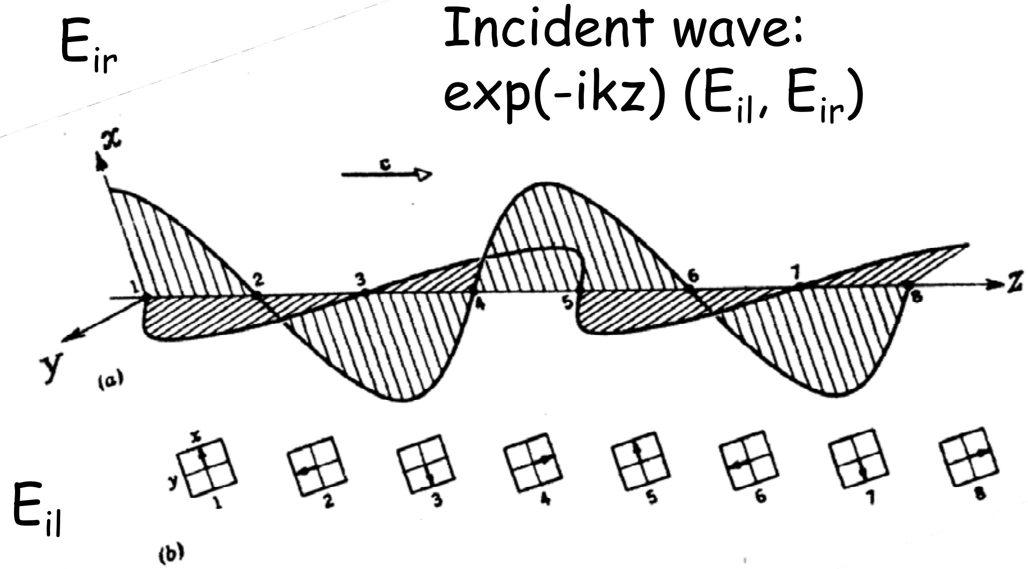
Scattering in the forward corresponds to $\Delta\Phi=0$, always constructively add

Larger the particle (more dipoles and the larger is $2\pi r/\lambda$), the larger is the forward scattering

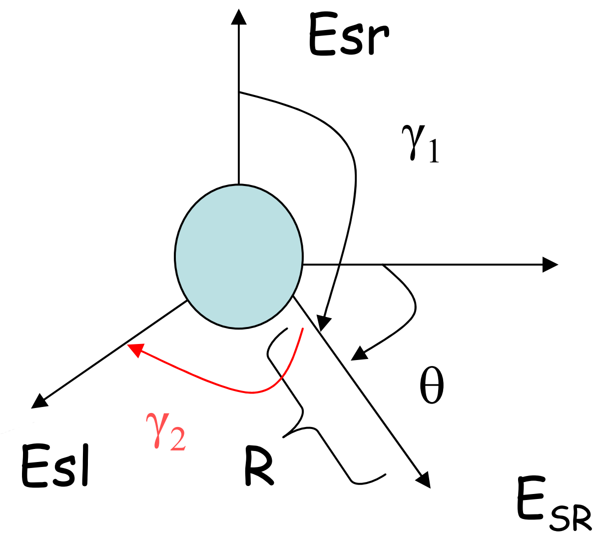
The more larger is $2\pi r/\lambda$, the more convoluted (greater # of max-min) is the scattering pattern

Scattering by a single particle

Incident wave:
exp(-ikz) (E_{il}, E_{ir})



Scattering by a particle



$$\begin{bmatrix} E_{sr} \\ E_{sl} \end{bmatrix} = \frac{\exp(-ikR + ikz)}{ikR} \begin{bmatrix} S_2(\theta) & S_3(\theta) \\ S_4(\theta) & S_1(\theta) \end{bmatrix} \begin{bmatrix} E_{il} \\ E_{ir} \end{bmatrix}$$

For spherical particles, S3 and S4 are equal zero. The scattering problem is then to find analytical expression of S2 and S1 by using electromagnetic theory, which was done by Lorentz in 1890 and Mie in 1908.

Lorentz-Mie theory

Angular distribution function:

$$S_1(\Theta) = \sum_{n=1}^{\infty} \frac{2n+1}{n(n+1)} \left[a_n \pi_n(\cos\ \Theta) + b_n \tau_n(\cos\ \Theta) \right]$$

$$S_2(\Theta) = \sum_{n=1}^{\infty} \frac{2n+1}{n(n+1)} \left[b_n \pi_n(\cos\ \Theta) + a_n \tau_n(\cos\ \Theta) \right]$$

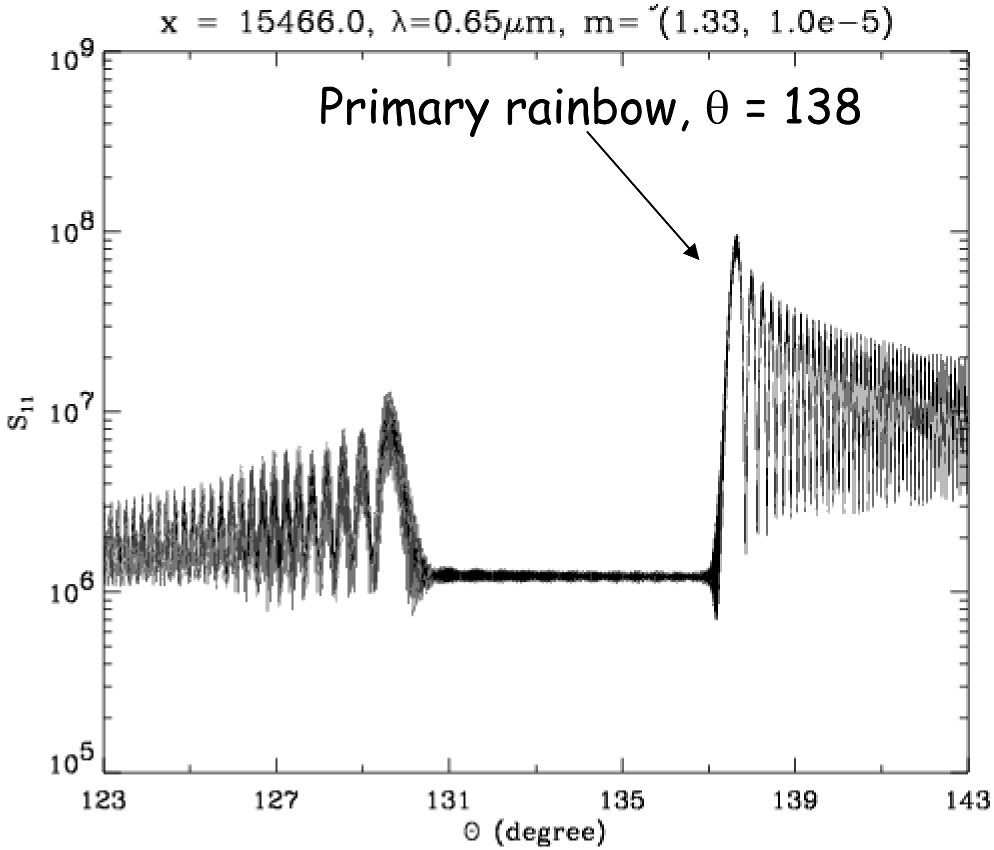
$$\pi_n(\cos\ \Theta) = \frac{1}{\sin(\Theta)} P_n^1(\cos\ \Theta)$$

Associated Legendre polynomial

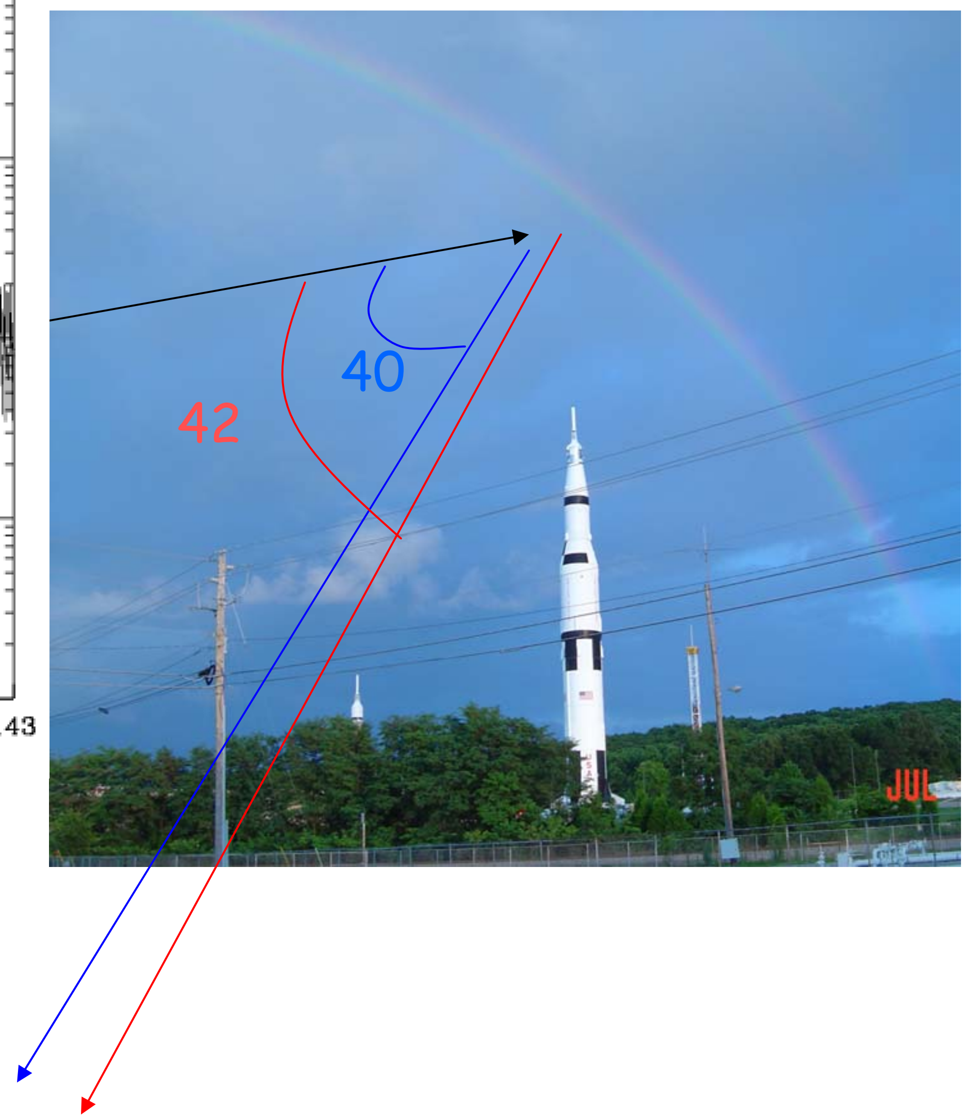
$$\tau_n(\cos\ \Theta) = \frac{d}{d\Theta} P_n^1(\cos\ \Theta)$$

Near field

Rainbow



x = 15466.0, λ=0.65μm, m=˝(1.33, 1.0e−5)
Primary rainbow, θ = 138
S_{11}
10^9
10^8
10^7
10^6
10^5
123
127
131
135
139
143
Θ (degree)



40
42

Scattering Properties (far field)

Extinction cross section:

$$\sigma_e = \frac{4\pi}{k^2} \mathrm{Re}[S_{1,2}(0^0)] \qquad S_1(0^0) = S_2(0^0) = \frac{1}{2}\sum_{n=1}^{\infty}(2n+1)(a_n + b_n)$$

Extinction Efficiency: (x is the size parameter)

$$Q_e = \frac{\sigma_e}{\pi a^2} \qquad Q_e = \frac{2}{x^2}\sum_{n=1}^{\infty}(2n+1)\,\mathrm{Re}[a_n + b_n]$$

Scattering Efficiency:

$$Q_s = \frac{2}{x^2}\sum_{n=1}^{\infty}(2n+1)[|a_n|^2 + |b_n|^2]$$

Absorption Efficiency:

$$Q_a = Q_e - Q_s$$

Single Scattering Albedo

$$\omega = Q_s/Q_e$$

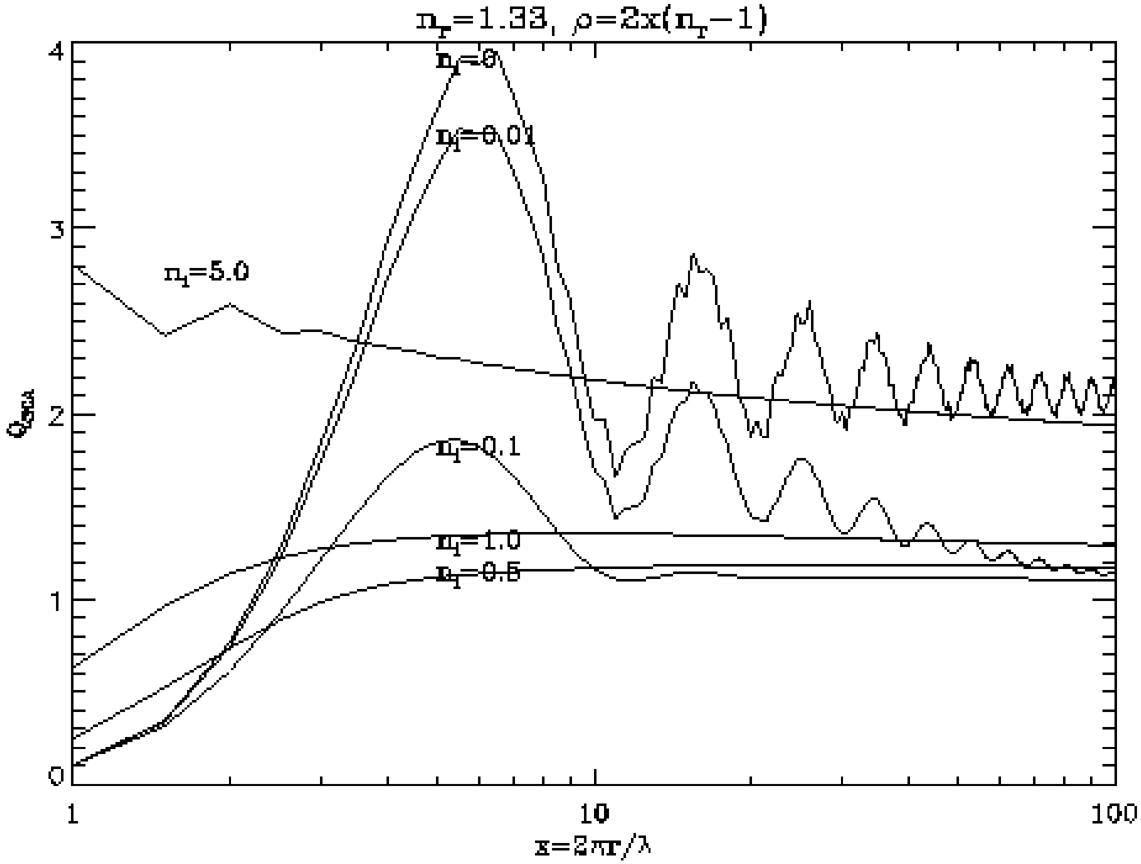
Physical meaning of extinction cross section

The area C_{ext} that, when multiplied by the irradiance of electromagnetic waves incident on an object, gives the total radiant flux scattered and absorbed by the object.

Similarly C_{sca}, $C_{abs.}$ The efficiency factor then follows

$$Q_{ext,sca,abs} = \frac{C_{ext,sca,abs}}{\pi r^2}$$

When size parameter becomes larger, Q_{ext} = 2.



Stokes Parameter

A set of four parameters was first introduced by Stokes (1852) to better characterize the light and interpret the light transfer

$$I = E_l E_l^* + E_r E_r^*$$

Intensity

$$Q = E_l E_l^* - E_r E_r^*$$

Degree of polarization

$$U = E_l E_r^* + E_r E_l^*$$

Plane of polarization

$$V = -i(E_l E_r^* - E_r E_l^*)$$

The ellipticity

For single wave,

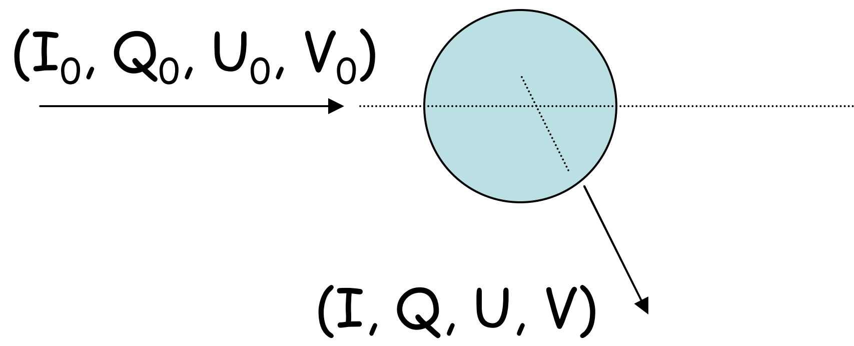
$I = E_l^2 + E_r^2$
$Q = E_l^2 - E_r^2$
$U = 2a_l\, a_r \cos(\Delta\phi)$
$V = 2a_l\, a_r \sin(\Delta\phi)$
$I^2 = Q^2 + U^2 + V^2$

Note, the actual light consists of many waves with different phases. For a measurement or detector, its measured light intensity is the result of many waves averaged over a certain amount of time. In this case, it can be proved:

$I^2 >= Q^2 + U^2 + V^2$.
Degree of polarization: $\mathrm{sqrt}(Q^2 + U^2 + V^2)/I$
Linear polarization $= -Q/I = -(I_l - I_r)/(I_l + I_r)$

Scattering matrix

$$\begin{bmatrix} I \\ Q \\ U \\ V \end{bmatrix} = \frac{\sigma_s}{4\pi r^2} P \begin{bmatrix} I_o \\ Q_o \\ U_0 \\ V_o \end{bmatrix}$$



P: scattering matrix. In general, P is a 4X4 matrix consisting of 16 different elements. For spherical and homogenous particles (Lorentz-Mie theory), P =

$$\begin{bmatrix} P_{11} & P_{12} & & \\ P_{12} & P_{11} & & \\ & & P_{33} & -P_{34} \\ & & P_{34} & P_{33} \end{bmatrix}$$

$$P_{11} = \frac{4\pi}{2k^2\sigma_s}\left[S_1 S_1^* + S_2 S_2^*\right]$$

$$P_{12} = \frac{4\pi}{2k^2\sigma_s}\left[S_2 S_2^* - S_1 S_1^*\right]$$

$$P_{33} = \frac{4\pi}{2k^2\sigma_s}\left[S_2 S_1^* + S_1 S_2^*\right]$$

$$-P_{34} = \frac{4\pi}{2k^2\sigma_s}\left[S_1 S_2^* - S_2 S_1^*\right]$$

The term "phase function" generally refers to P_{11}.

Scattering Properties of an ensemble of particles

To model the atmospheric radiative transfer, the overall (bulk) scattering properties of an ensemble of particles are needed. In particle, the aerosol size distribution is described by an analytical forma (such as lognormal or gamma distribution) to facilitate the computation of bulk scattering properties.

m^{-1} m^2 $\#/m^3$

$$\beta_e = \int_{r_{min}}^{r_{max}} \sigma_e(r) N(r) dr$$

$$\beta_s = \int_{r_{min}}^{r_{max}} \sigma_s(r) N(r) dr$$

$$\beta_a = \int_{r_{min}}^{r_{max}} \sigma_a(r) N(r) dr$$

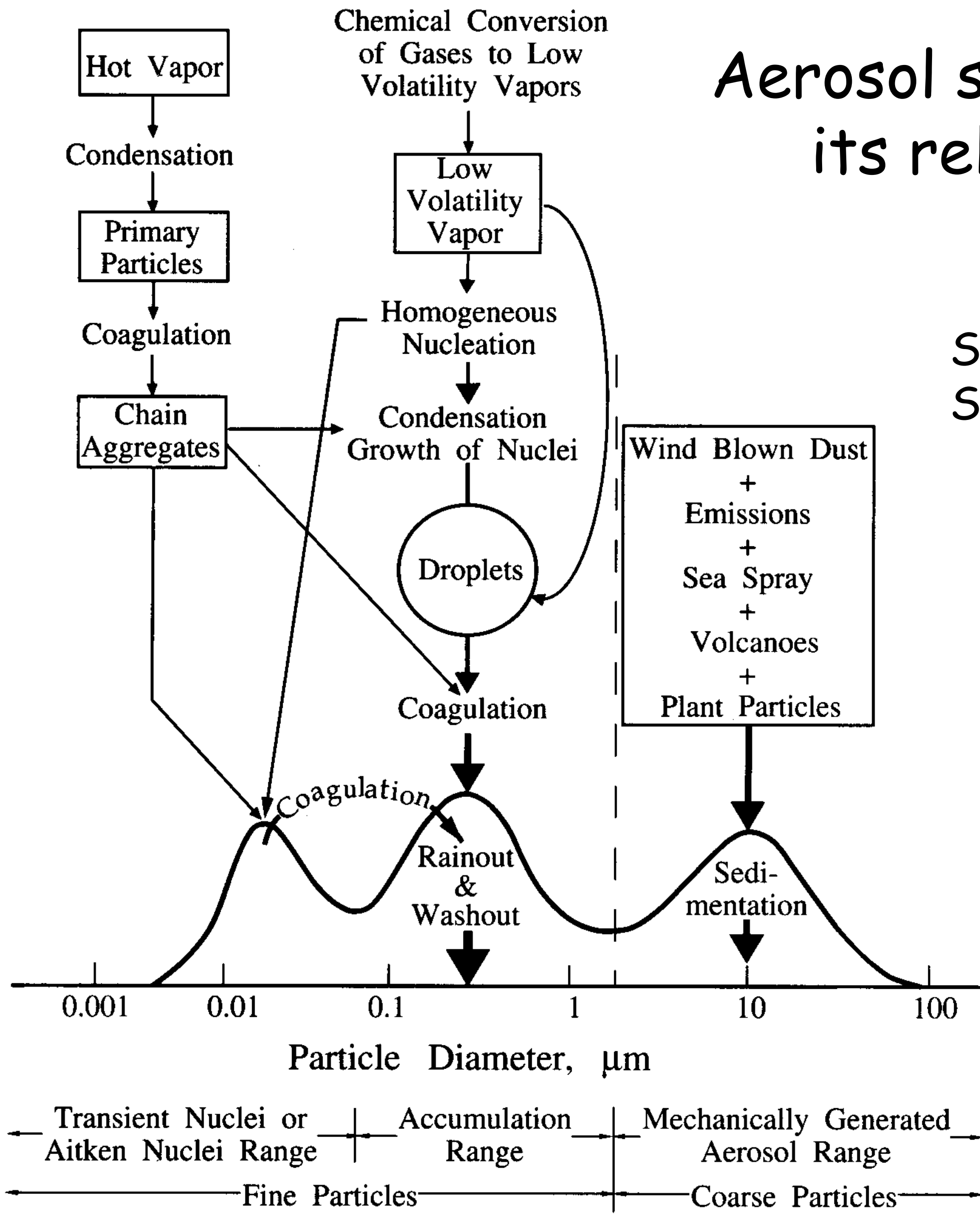
L-M calculation Particle size distribution

$$P(\Theta) = \frac{\int_{r_{min}}^{r_{max}} P_r(\Theta) \sigma_s N(r) dr}{\beta_s}$$

$$g = \frac{1}{2} \int_{-1}^{1} P(\cos\Theta) \cos(\Theta) d(\cos\Theta)$$

Optical thickness $\tau = \int \beta(z)\, dz$

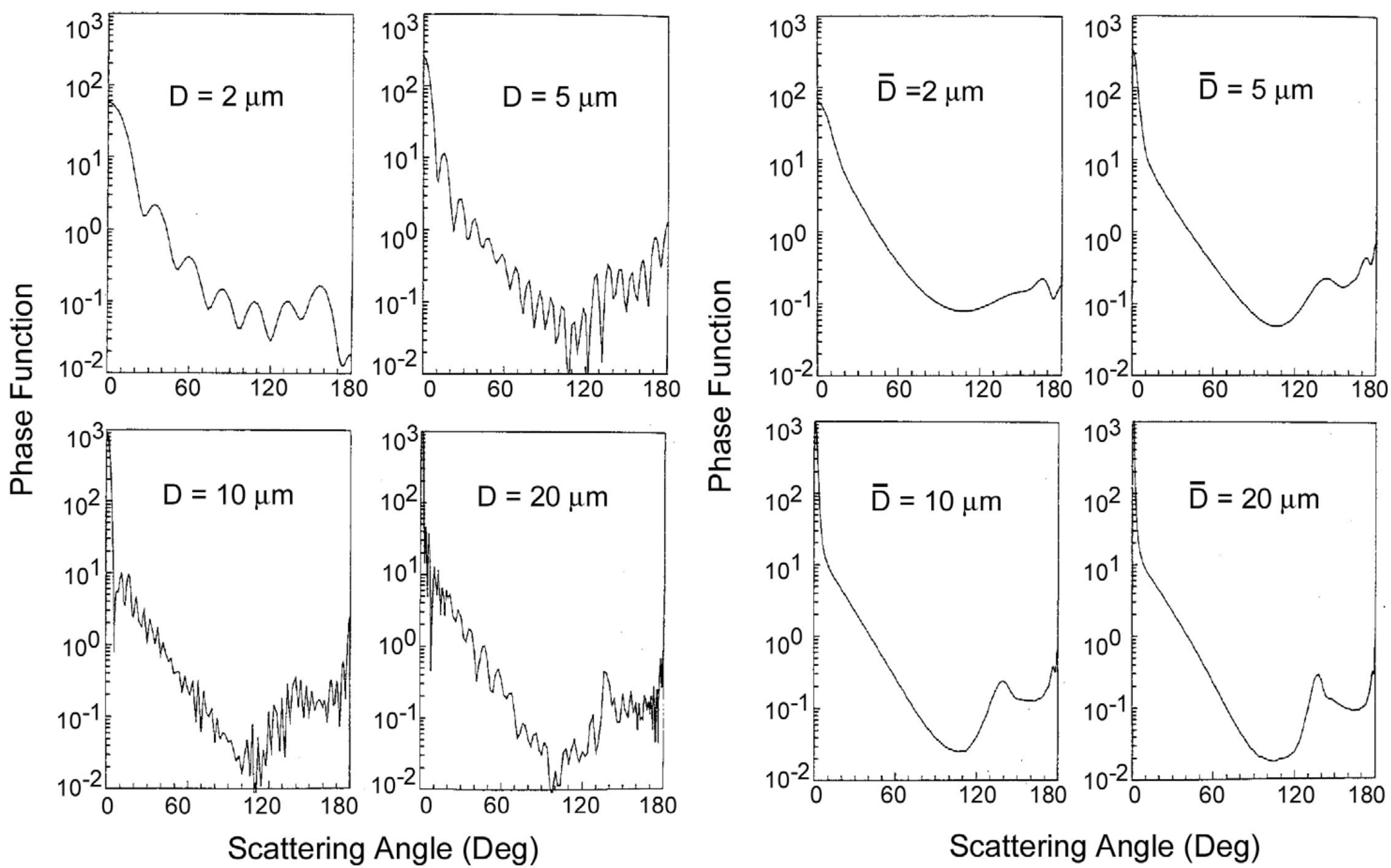
Aerosol size distribution and its relevant processes



Sea salt & Dust, > 1um
Smoke & sulfate 0.1 – 0.2um

Phase function of aerosols

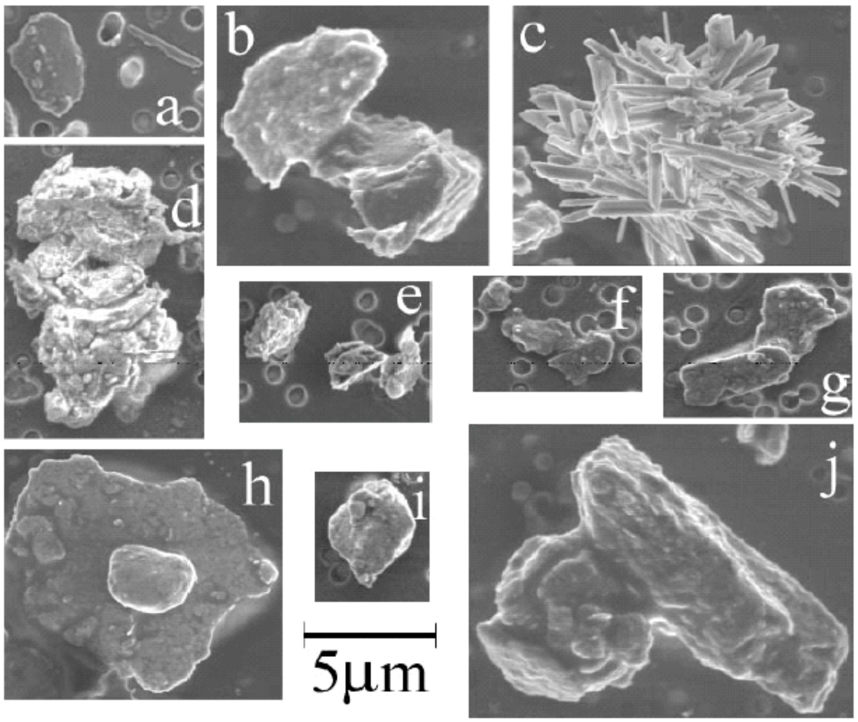
Single particle (visible) An ensemble of particles

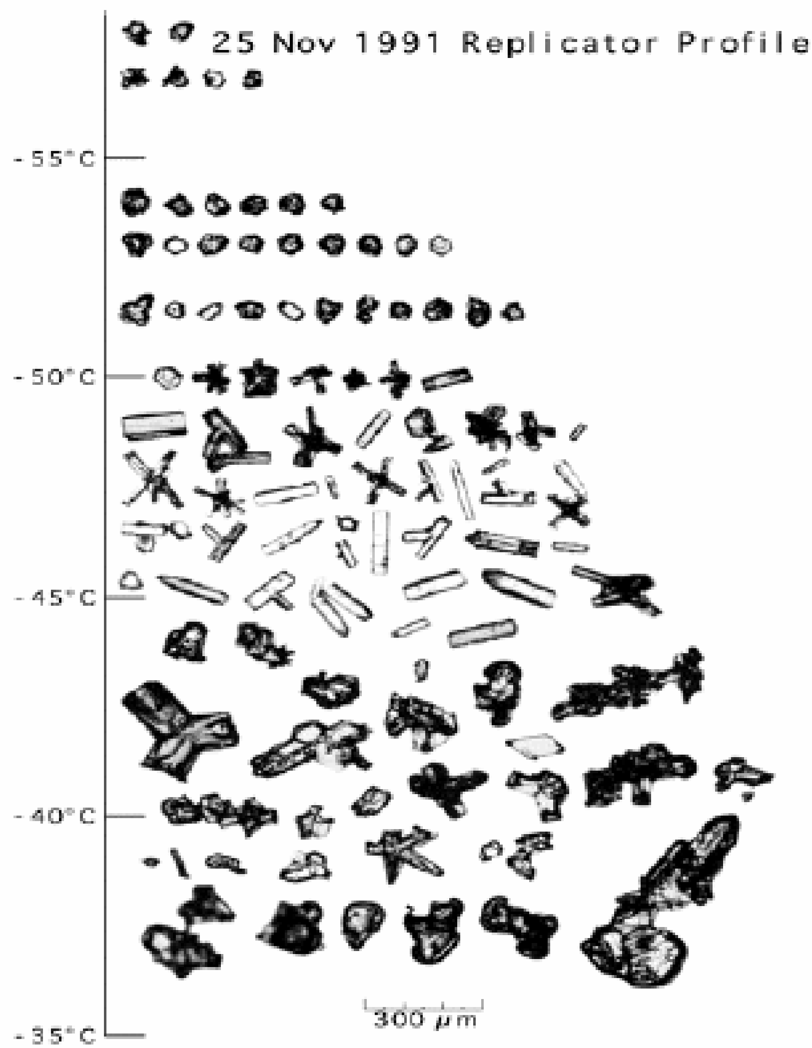


Non-spherical particles

Ice crystal profile

Saharan dust particles collected in Puerto Rico





L-M theory can not be applied to non-spherical particles.

Techniques for computing scattering properties of non-spherical particles

16 different elements !

$$P = \begin{bmatrix} P_{11} & P_{12} & P_{13} & P_{14} \\ P_{21} & P_{22} & P_{23} & P_{24} \\ P_{31} & P_{32} & P_{33} & P_{34} \\ P_{41} & P_{42} & P_{43} & P_{44} \end{bmatrix}$$

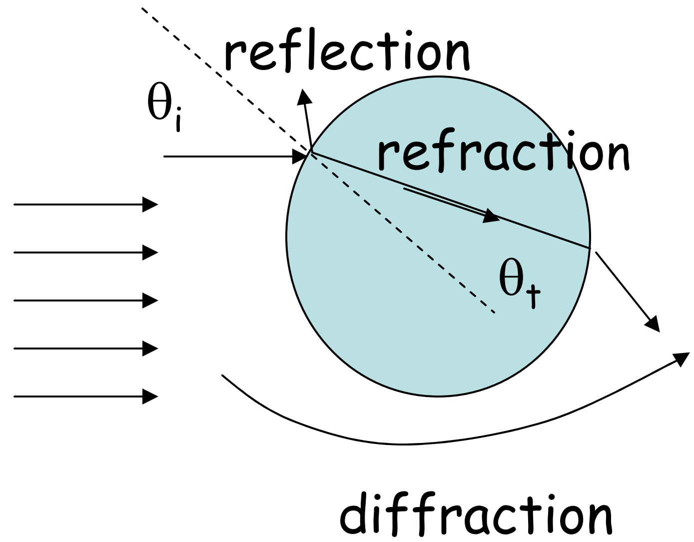
Methods:

1) Ray-tracing (geometric optics)
2) T-matrix
3) FDTD (finite difference time domain)
4) Discrete dipole approximation

In contrast to spherical particles, the particle shape and the particle orientation to the incident light play an important role in determining the scattering properties, particularly, the phase function.

Ray-tracing

- For size parameter > 100
- Incident EM consists of a collection of parallel rays
- Fresnel reflectance and transmission formula applied to each ray
- Diffraction method is used for the peak in forward scattering
- Monte Carlo approach is used to simulate the whole scattering process



Advantages: any shape

Dis-advantages: size limitation (x should be larger), not an exact solution, other treatment is needed for absorbing particles

Snell's law:

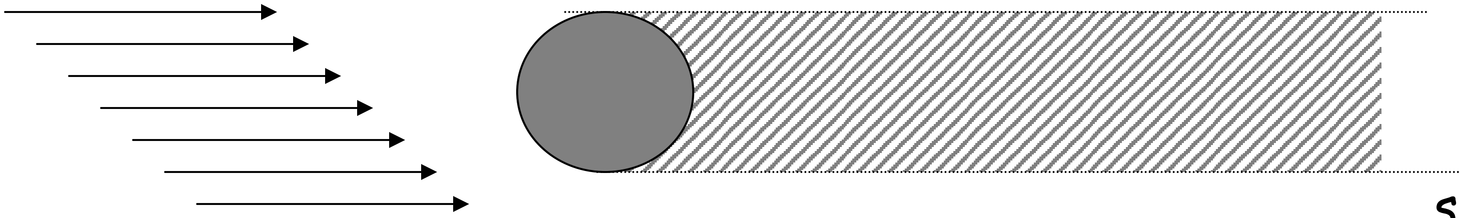
$$\sin \theta_i = m \sin \theta_t$$

Fresnel reflectance:

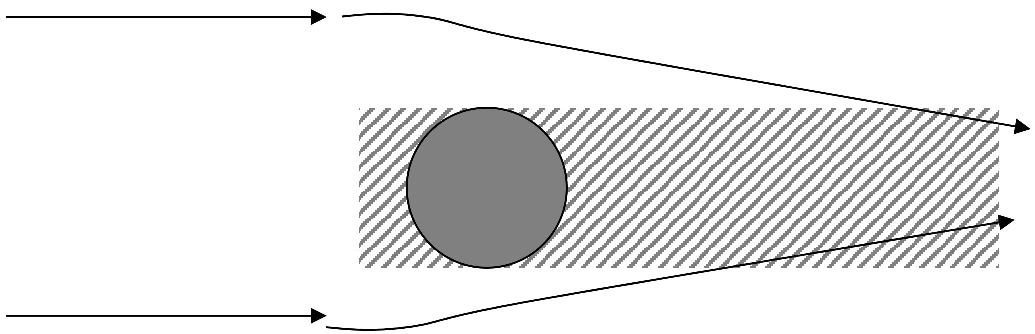
$$r_r = \frac{\cos\theta_i - \sqrt{m^2 - \sin^2\theta_i}}{\cos\theta_i + \sqrt{m^2 - \sin^2\theta_i}}$$

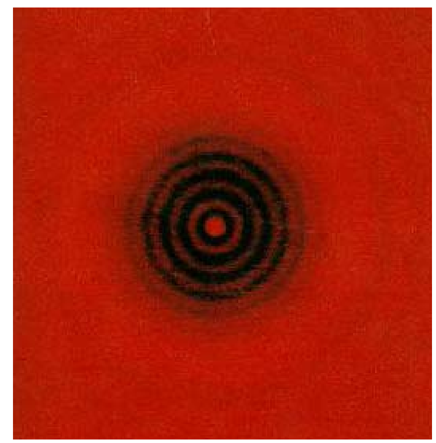
$$r_l = \frac{\sqrt{m^2 - \sin^2\theta_i} - m^2\cos\theta_i}{\sqrt{m^2 - \sin^2\theta_i} + m^2\cos\theta_i}$$

Extinction Paradox



$$Q_{ext} = \frac{\text{shadow area}}{\pi r^2} = 1\,??$$





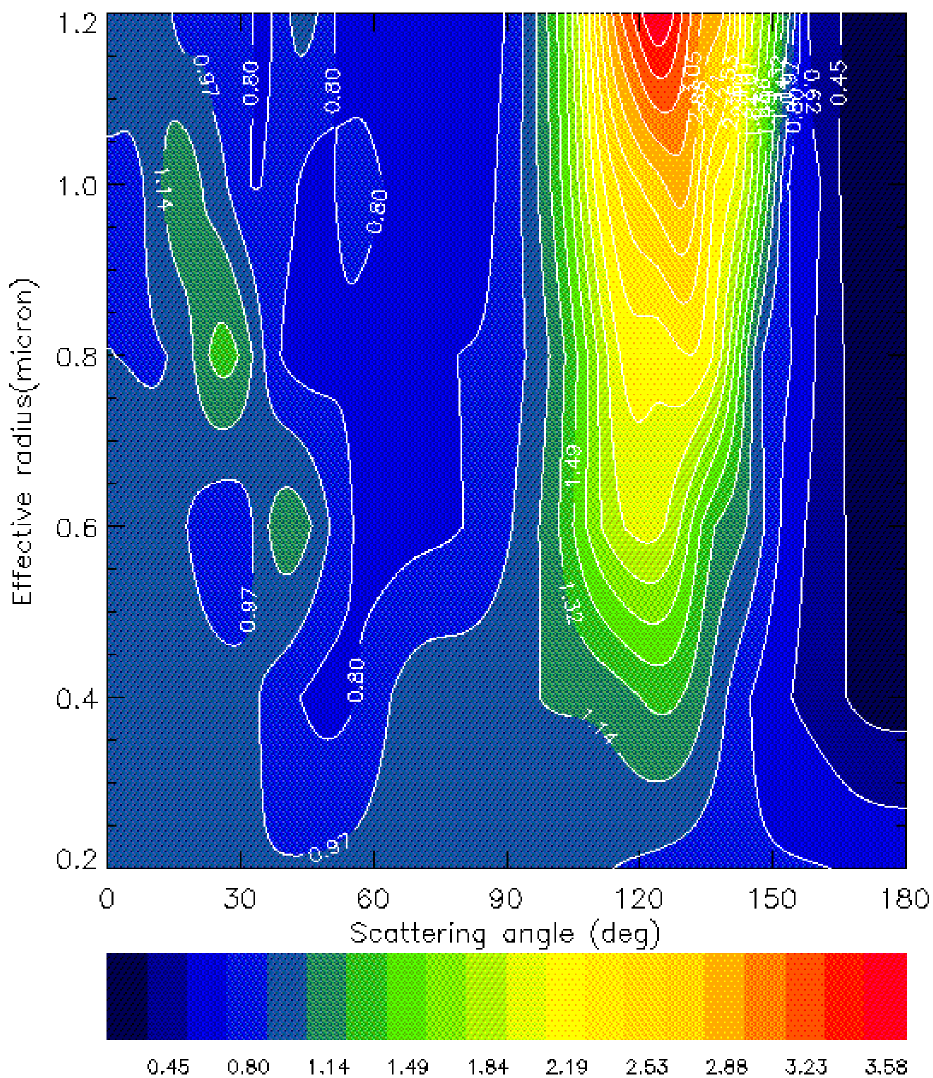
Poisson / Arago spot

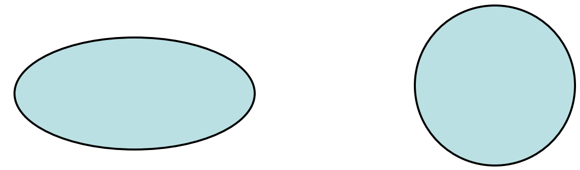
$$Q_{ext} = \frac{\text{shadow area by reflectaion and absorption} + \text{area filled by diffraction}}{\pi r^2}$$

$$= \frac{\pi r^2 + \pi r^2}{\pi r^2} = 2$$

Poisson originally predicted the existence of such a spot. His original motivation is to disprove the wave theory, since such a spot is a counterintuitive result. However, Arago later observed such a spot, which proves the wave nature of light

Phase function ratio between spherical and non-spherical particles





With same surface area, spheroids shows larger phase function for $90<\theta<120$ and smaller P for $150<\theta<180$.

Calculation with T-matrix codes.

http://www.giss.nasa.gov/~crmim/t_matrix.html

Further Reading

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